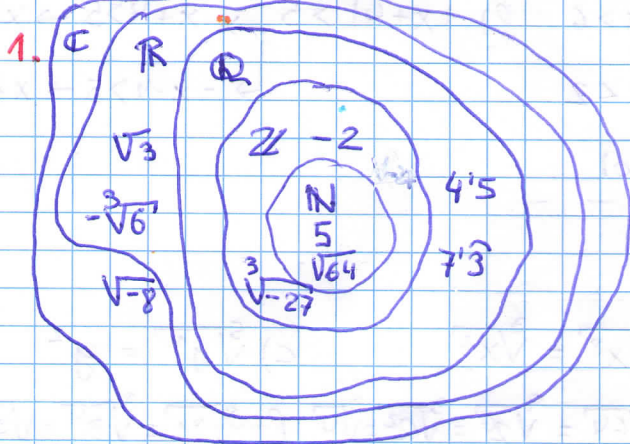


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El's nombres reales - da recta real



- $N \rightarrow 5, \sqrt{64}$
- $Z \rightarrow -2, \sqrt[3]{-27}, N$
- $Q \rightarrow 4/5, 7/3, Z$
- $R \rightarrow \sqrt{3}, -\sqrt[3]{6}, Q$
- $C \rightarrow \sqrt{-8}$

- 2. NATURALS, N $5, \sqrt{64}$
- ENTERS, Z $5, \sqrt{64}, -2, \sqrt[3]{-27}$
- RACIONALS, Q $5, \sqrt{64}, -2, 4/5, 7/3, \sqrt[3]{-27}$
- REALS, R $5, \sqrt{64}, -2, 4/5, 7/3, \sqrt{3}, -\sqrt[3]{6}, \sqrt[3]{-27}$
- NO REALS $\sqrt{-8}$

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- 3. a) $(-3, -1)$ $-3 < x < -1$ b) $[4, +\infty)$ $4 \leq x$
- c) $(3, 9]$ $3 < x \leq 9$ d) $(-\infty, 0)$ $x < 0$

- 4. a) $\{x | -2 \leq x < 5\}$ $[-2, 5)$
- b) $[-2, 5) \cup (5, 7]$
- c) $(-\infty, 0) \cup (3, +\infty)$
- d) $(-\infty, 1) \cup (1, +\infty)$

- 5. a) $|-11| = 11$ b) $|\pi| = \pi$ c) $|\sqrt{5}| = \sqrt{5}$
- d) $|0| = 0$ e) $|3 - \pi| = \pi - 3$ f) $|3 - \sqrt{2}| = 3 - \sqrt{2}$
 $\sqrt{2} = 1.4142...$
- g) $|1 - \sqrt{2}| = \sqrt{2} - 1$ h) $|\sqrt{2} - \sqrt{3}| = \sqrt{3} - \sqrt{2}$ i) $|7 - \sqrt{50}| = \sqrt{50} - 7$
 $\sqrt{50} = 7.07...$

6. a) $|x| = 5 \quad x = -5, x = 5$

b) $|x| \leq 5 \Leftrightarrow -5 \leq x \leq 5 \Leftrightarrow x \in [-5, 5]$

$\begin{cases} x \leq 5 \\ -x \leq 5 \rightarrow x \geq -5 \end{cases}$

c) $|x-4| = 2$

$\rightarrow x-4=2 \rightarrow x=6$
 $\rightarrow 4-x=2 \rightarrow x=2$

d) $|x-4| \leq 2 \Rightarrow -2 \leq x-4 \leq 2 \Rightarrow$
 $\Rightarrow 2 \leq x \leq 6 \Leftrightarrow x \in [2, 6]$

e) $|x-4| > 2$

$\rightarrow x-4 > 2 \rightarrow x > 6$
 $\rightarrow 4-x > 2 \rightarrow x < 2$

f) $|x+4| > 5 \rightarrow x+4 > 5 \rightarrow x > 1$

$\rightarrow -x-4 > 5 \rightarrow x < -9$

$(-\infty, 2) \cup (6, +\infty)$

Página 15 RADICALS

7. a) $\sqrt[12]{x^9} = \sqrt{x^3}$

b) $\sqrt[12]{x^8} = \sqrt[3]{x^2}$

c) $\sqrt[5]{y^{10}} = y^2$

d) $\sqrt[6]{8^1} = \sqrt[6]{2^3} = \sqrt{2}$

e) $\sqrt[9]{6^4} = \sqrt[9]{6^3} = \sqrt[3]{22} = \sqrt[3]{4}$

f) $\sqrt[8]{8^1} = \sqrt[8]{8^4} = \sqrt{3}$

8. $\sqrt[3]{3^1} > \sqrt[3]{1^3} \Leftrightarrow$

$\sqrt[3]{3^1} = \sqrt[12]{3^4} = \sqrt[12]{2^4 \cdot 3^4}$

$\sqrt[3]{1^3} = \sqrt[12]{1^4} = \sqrt[12]{2^0 \cdot 3^0}$

9. a) $\sqrt[12]{a^5} = \sqrt[36]{a^{15}}$

$\sqrt[12]{a^3} = \sqrt[36]{a^{14}}$

b) $\sqrt[3]{5^1} = \sqrt[9]{5^3} = \sqrt[9]{13265^1}$

$\sqrt[9]{132650} = \sqrt[9]{132650}$

mcdm(12, 18) = $2^2 \cdot 3^2 = 4 \cdot 9 = 36$

mcm(m(3, 9)) = 3^2

$12 = 2^2 \cdot 3$
 $18 = 2 \cdot 3^2$

10. a) $(\sqrt{\sqrt{k}})^8 = (\sqrt[8]{k})^8 = k$

b) $\sqrt[5]{3 \cdot x^{10}} = \sqrt[15]{x^{10}} = \sqrt[3]{x^2}$

c) $\sqrt[3]{(\sqrt{x})^6} = \sqrt[6]{x^6} = x$

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11. a) $\sqrt[3]{2^1} \cdot \sqrt[5]{2} = \sqrt[15]{2^5} \cdot \sqrt[15]{2^3} = \sqrt[15]{2^8} = \sqrt[15]{2^8}$

b) $\sqrt[9]{9^1} \cdot \sqrt[6]{3} = \sqrt[9]{9^2} \cdot \sqrt[6]{3} = \sqrt[6]{(3^2)^2 \cdot 3} = \sqrt[6]{3^5}$

c) $\sqrt{2^1} \cdot \sqrt[4]{2} \cdot \sqrt[8]{2} = \sqrt[2^4]{2^4} \cdot \sqrt[2^4]{2^2} \cdot \sqrt[2^4]{2^1} = \sqrt[2^4]{2^7}$

d) $\sqrt[4]{8^1} \cdot \sqrt[3]{9^1} = \sqrt[12]{8^3} \cdot \sqrt[12]{9^4} = \sqrt[12]{2^3 \cdot 3^3} \cdot \sqrt[12]{12^4} = \sqrt[12]{2^9 \cdot 3^8} = \sqrt[12]{2^9 \cdot 2^8} = \sqrt[12]{2^{17}} = \sqrt[12]{2^{17}}$

12. a) $\sqrt[3]{x^1} = \sqrt[15]{x^3} = \sqrt[15]{x^3}$

b) $\sqrt[3]{a \cdot b} = \sqrt[6]{a^2 \cdot b^2} = \sqrt[6]{a^2 \cdot b^2}$

c) $\frac{\sqrt[6]{a^3}}{\sqrt[3]{a^2}} = \sqrt[6]{\frac{a^3}{a^4}} = \sqrt[6]{\frac{1}{a}} = \sqrt[6]{a^{-1}} = \frac{\sqrt[6]{a^3 \cdot b^5 \cdot c}}{\sqrt[6]{a^2 \cdot b^2}} = \frac{\sqrt[6]{3 \cdot 5 \cdot c}}{\sqrt[6]{2 \cdot 5}} = \frac{1^4 \sqrt[6]{a}}{c \sqrt[6]{5 \cdot c}}$

$$\begin{array}{r} 729 \ 3 \\ 243 \ 3 \\ 81 \ 3 \\ 27 \ 3 \\ 9 \ 3 \\ 3 \ 3 \\ 1 \end{array}$$

$$729 = 3^6$$

13. a) $\frac{\sqrt[3]{3^2}}{\sqrt{3}} = \sqrt{\frac{3^4}{3^3}} = \sqrt{3}$ b) $\frac{\sqrt{9}}{\sqrt[3]{3}} = \sqrt{\frac{9^3}{3^2}} = \sqrt{\frac{(3^2)^3}{3^2}} = \sqrt{\frac{3^6}{3^2}} = \sqrt{3^4} = 3^2 = 9$

c) $\frac{\sqrt[5]{16}}{\sqrt{2}} = \sqrt{\frac{(2^4)^2}{2^5}} = \sqrt{\frac{2^8}{2^5}} = \sqrt{2^3} = \sqrt{2^2 \cdot 2} = 2\sqrt{2}$ d) $\frac{\sqrt[4]{729}}{\sqrt{3}} = \sqrt{\frac{3^6}{3^2}} = \sqrt{3^4} = 3^2 = 9$

14. a) $5\sqrt{x} + 3\sqrt{x} + 2\sqrt{x} = 10\sqrt{x}$

b) $\sqrt{9 \cdot 2} + \sqrt{25 \cdot 2} - \sqrt{2} = \sqrt{3^2 \cdot 2} + \sqrt{5^2 \cdot 2} - \sqrt{2} = 3\sqrt{2} + 5\sqrt{2} - \sqrt{2} = 7\sqrt{2}$

c) $\sqrt{18} + \sqrt{50} - \sqrt{2} - \sqrt{8} = \sqrt{3^2 \cdot 2} + \sqrt{2 \cdot 5^2} - \sqrt{2} - \sqrt{2^3} = 3\sqrt{2} + 5\sqrt{2} - \sqrt{2} - 2\sqrt{2} = 5\sqrt{2}$

d) $\sqrt{27} - \sqrt{50} + \sqrt{12} + \sqrt{8} = \sqrt{3^3} - \sqrt{2 \cdot 5^2} + \sqrt{2^2 \cdot 3} + \sqrt{2^3} = 3\sqrt{3} - 5\sqrt{2} + 2\sqrt{3} + 2\sqrt{2} = 5\sqrt{3} - 3\sqrt{2}$

e) $\sqrt{50a} - \sqrt{18a} = \sqrt{2 \cdot 5^2 \cdot a} - \sqrt{2 \cdot 3^2 \cdot a} = 5\sqrt{2a} - 3\sqrt{2a} = 2\sqrt{2a}$

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15. a) $\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$ b) $\frac{3}{\sqrt[3]{4}} = \frac{3}{\sqrt[3]{2^2}} = \frac{3\sqrt[3]{2}}{2}$

c) $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{7 \cdot 2}}{2} = \frac{\sqrt{14}}{2}$ d) $\frac{1}{\sqrt{a^3}} = \frac{1}{a\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a^2}$

e) $\frac{3}{\sqrt{50}} = \frac{3}{\sqrt{50}} \cdot \frac{\sqrt{50}}{\sqrt{50}} = \frac{3\sqrt{50}}{50} = \frac{3\sqrt{2}}{10}$ f) $\frac{4}{\sqrt{18}} = \frac{4}{\sqrt{18}} \cdot \frac{\sqrt{18}}{\sqrt{18}} = \frac{4\sqrt{18}}{18} = \frac{2\sqrt{18}}{9} = \frac{2\sqrt{2}}{3}$

g) $\frac{2}{\sqrt[3]{25}} = \frac{2}{\sqrt[3]{5^2}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{2\sqrt[3]{5}}{5}$ h) $\frac{1}{\sqrt[3]{40}} = \frac{1}{2\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{\sqrt[3]{5}}{2 \cdot 5} = \frac{\sqrt[3]{5}}{10}$

i) $\frac{3}{\sqrt[3]{36}} = \frac{3}{\sqrt[3]{6^2}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}} = \frac{3\sqrt[3]{6}}{6} = \frac{\sqrt[3]{6}}{2}$ j) $\frac{2}{\sqrt[3]{100}} = \frac{2}{\sqrt[3]{10^2}} \cdot \frac{\sqrt[3]{10}}{\sqrt[3]{10}} = \frac{2\sqrt[3]{10}}{10} = \frac{\sqrt[3]{10}}{5}$

16. a) $\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

b) $\frac{x+y}{\sqrt{x}+\sqrt{y}} = \frac{x+y}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{(x+y)(\sqrt{x}-\sqrt{y})}{x-y} = \frac{x\sqrt{x}-x\sqrt{y}+y\sqrt{x}-y\sqrt{y}}{x-y}$

c) $\frac{a-1}{\sqrt{a}-1} = \frac{a-1}{\sqrt{a}-1} \cdot \frac{\sqrt{a}+1}{\sqrt{a}+1} = \frac{(a-1)(\sqrt{a}+1)}{a-1} = \sqrt{a}+1$

d) $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{(\sqrt{x}+\sqrt{y})^2}{x-y} = \frac{x+y+2\sqrt{xy}}{x-y}$

e) $\frac{1}{2\sqrt{3}-\sqrt{5}} = \frac{1}{2\sqrt{3}-\sqrt{5}} \cdot \frac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}} = \frac{2\sqrt{3}+\sqrt{5}}{4 \cdot 3 - 5} = \frac{2\sqrt{3}+\sqrt{5}}{12-5} = \frac{2\sqrt{3}+\sqrt{5}}{7}$

$$f) \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \cdot \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{(3\sqrt{2}+2\sqrt{3})^2}{9 \cdot 2 + 4 \cdot 3} = \frac{9 \cdot 2 + 4 \cdot 3 + 12\sqrt{6}}{18 + 12} = \frac{18 + 12 + 12\sqrt{6}}{30} = \frac{30 + 12\sqrt{6}}{30} = 5 + 2\sqrt{6}$$

$$g) \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}}{2} + \sqrt{2} + 1 + \sqrt{2} - 1 = \frac{\sqrt{2}}{2} + 2\sqrt{2} = \frac{\sqrt{2} + 4\sqrt{2}}{2} = \frac{5\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \left| \quad = \frac{\sqrt{2} + 4\sqrt{2}}{2} = \frac{5\sqrt{2}}{2} \right.$$

$$\frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$$

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

$$h) \frac{1}{\sqrt{x}-\sqrt{y}} + \frac{1}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{x-y} + \frac{\sqrt{x}-\sqrt{y}}{x-y} = \frac{\sqrt{x}+\sqrt{y}+\sqrt{x}-\sqrt{y}}{x-y} = \frac{2\sqrt{x}}{x-y}$$

$$\frac{1}{\sqrt{x}-\sqrt{y}} = \frac{1}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{x-y} \quad \left| \quad = \frac{2\sqrt{x}}{x-y} \right.$$

$$\frac{1}{\sqrt{x}+\sqrt{y}} = \frac{1}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$$

Pàg 18. Nombres aproximats. Notació científica

17. Valor aproximat de $\pi = 3,14$

Error absolut = valor real - valor aproximat = $\pi - 3,14 < 0,005$

Error relatiu = $\frac{\text{valor absolut}}{\text{valor aprox}} = \frac{0,005}{3,14} < 0,0016$

18. 100ϕ amb 6 xifres significatives

$$100 \phi = 161,803399 \quad \frac{1+\sqrt{5}}{2} = \phi$$

Acotan l'error absolut $< 0,0000005$

Acotan l'error relatiu $< \frac{0,0000005}{161,803399} < 3,09 \cdot 10^{-9}$

19. a) $1,49 \cdot 10^8$ Km

b) $1,49 \cdot 10^{13}$ cm $\rightarrow 1,5 \cdot 10^{13}$ cm

c) Acotan l'error absolut Km $\left\{ \begin{array}{l} E_a < 500.000 = 0,005 \cdot 10^8 \\ E_r < \frac{0,005}{1,49} < 0,003 \end{array} \right.$ cm $\left\{ \begin{array}{l} E_a < 5 \cdot 10^{11} \\ E_r < \frac{0,05}{1,5} < 0,03 \end{array} \right.$

Pàg 23 LOGARITMES

20. a) $\log_2 16 = 4$ $2^x = 16 \rightarrow 2^x = 2^4$

b) $\log_2 0.125 = -2$ $2^x = 0.125 \rightarrow 2^x = 2^{-2}$ $(0.125 = \frac{25}{100} = \frac{1}{4} = \frac{1}{2^2} = 2^{-2})$

c) $\log_9 1 = 0$ $9^x = 1 \rightarrow 9^x = 9^0$

d) $\log_{10} 0.1 = -1$ $10^x = 0.1 \rightarrow 10^x = 10^{-1}$

e) $\log_{34} 64 = 3$ $4^x = 64 \rightarrow 4^x = 4^3$

f) $\log_7 49 = 2$ $7^x = 49 \rightarrow 7^x = 7^2$

g) $\ln e^4 = 4$ $e^x = e^4$

$(0.04 = \frac{4}{100} = \frac{2}{50} = \frac{1}{25} = \frac{1}{5^2} = 5^{-2})$

h) $\ln e^{-114} = -114$ $e^x = e^{-114}$

↑

i) $\log_5 0.04 = -2$ $5^x = 0.04 \rightarrow 5^x = \frac{1}{5^2} \rightarrow 5^x = 5^{-2}$

j) $\log_6 \left(\frac{1}{216} \right) = -3$ $6^x = \frac{1}{6^3} \rightarrow 6^x = 6^{-3}$

$$\begin{array}{r} 216 \overline{) 6} \\ 36 \overline{) 6} \\ \underline{6} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

21. a) $\log_2 60$ $2^5 = 32$ i $2^6 = 64 \rightarrow 5 < \log_2 60 < 6$

b) $\log_5 700$ $5^4 = 625$ i $5^5 = 3125 \rightarrow 4 < \log_5 700 < 5$

c) $\log_{10} 43000$ $10^4 = 10000$ $10^5 = 100000 \rightarrow 4 < \log_{10} 43000 < 5$

d) $\log_{10} 0.084$ $10^{-2} = 0.01$ $10^{-1} = 0.1 \rightarrow -2 < \log_{10} 0.084 < -1$

e) $\log_9 60$ $9^1 = 9$ $9^2 = 81 \rightarrow 1 < \log_9 60 < 2$

f) $\ln e = 1$ $e^1 = e$

22. $\log_a P = \frac{\log P}{\log a}$

a) $\log_2 1500 = \frac{\log 1500}{\log 2} \approx 10.55$

b) $\log_3 200 = \frac{\log 200}{\log 3} \approx 3.129$

c) $\log_{100} 200 = \frac{\log 200}{\log 100} = 1.15$

d) $\log_{300} 40 = \frac{\log 40}{\log 300} = 0.180$

23. $\log_5 A = 1'8$ i. $\log_5 B = 2'4$

$$\begin{aligned} \text{a) } \log_5 \sqrt[3]{\frac{A^2}{25B}} &= \log_5 \left(\frac{A^2}{25B} \right)^{\frac{1}{3}} = \frac{1}{3} \log_5 \frac{A^2}{25B} = \\ &= \frac{1}{3} (\log_5 A^2 - \log_5 25B) = \frac{1}{3} (2 \log_5 A - \log_5 5^2 B) \\ &= \frac{1}{3} [2 \log_5 A - (\log_5 5^2 + \log_5 B)] = \\ &= \frac{1}{3} [2 \cdot \log_5 A - (2 + \log_5 B)] = \frac{1}{3} [2 \cdot \log_5 A - 2 - \log_5 B] \\ &= \frac{1}{3} [2 \cdot 1'8 - 2 - 2'4] = \frac{1}{3} [3'6 - 2 - 2'4] = \\ &= \frac{-0'8}{3} \approx -0'27 \end{aligned}$$

$$\begin{aligned} \bullet \log_5 \sqrt[3]{\frac{A^2}{25B}} &= \log_5 \left(\frac{A^2}{25B} \right)^{\frac{1}{3}} = \log_5 \frac{A^{\frac{2}{3}}}{(5^2 B)^{\frac{1}{3}}} = \\ &= \frac{2}{3} \log_5 A - \left(\frac{2}{3} \log_5 5 + \frac{1}{3} \log_5 B \right) = \\ &= \frac{2}{3} \log_5 A - \frac{2}{3} - \frac{1}{3} \log_5 B = \frac{2}{3} \cdot 1'8 - \frac{2}{3} - \frac{1}{3} \cdot 2'4 = \\ &= \frac{3'6 - 2 - 2'4}{3} \approx -0'27 \end{aligned}$$

b) $\log_5 \frac{5\sqrt{A^3}}{B^2} = \log_5 (5\sqrt{A^3}) - \log_5 B^2 = \log_5 5 + \log_5 \sqrt{A^3} -$
 $-\log_5 B^2 = 1 + \frac{3}{2} \log_5 A - 2 \log_5 B = 1 + \frac{3}{2} \cdot 1'8 - 2 \cdot 2'4 =$
 $= 1 + \frac{5'4}{2} - 4'8 = 1 + 2'7 - 4'8 = 3'7 - 4'8 = -1'1$

24. $\ln y = 2x - \ln 5 \rightarrow \ln y = \ln e^{2x} - \ln 5 \rightarrow \ln y = \ln \frac{e^{2x}}{5} \rightarrow$
 $\rightarrow y = \frac{e^{2x}}{5}$

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25. N $\rightarrow 2, 127$

Z $\rightarrow -13$

Q $\rightarrow 0'6, -\frac{5}{7}, \sqrt{\frac{16}{9}}, \frac{43}{13}$

R $\rightarrow \sqrt{3}, \pi$

26. $\frac{4}{7} \wedge \frac{5}{7}$

0'5314285714

0'314285714

Ex: rationally: $\frac{9}{14}, \frac{21}{35}, \frac{44}{70}$

irrational: $\sqrt{\frac{3x}{100}}$

27. a) $\frac{140}{99} < \sqrt{2}$ b) $0'52\overline{6} > 0'52\overline{6}$ c) $4'8\overline{9} > 2\sqrt{6}$ d) $-2'0\overline{98} > -2'1$

$\frac{140}{99} \approx 1'4142\dots$ $4'8\overline{9} \approx 4'898979\dots$

28. $-547 < \sqrt{8}$; $\frac{13}{3} > \frac{\sqrt{2}}{2}$; $\sqrt{4} > \frac{\pi}{2}$; $\frac{5}{12} < 0'34\overline{2}$

$\frac{13}{3} \approx 4'33\dots$ $\frac{\sqrt{2}}{2} \approx 0'707\dots$ $\frac{\pi}{2} \approx 1'57\dots$ $\frac{5}{12} \approx 0'416\dots$

29. Racional: $\frac{11}{7}, \frac{2}{3}$
 Irracional: $\frac{\sqrt{3}}{2}, 2\pi, e, \phi$

Potências

30. $\left(\frac{3}{2} - \frac{3}{4}\right)^{-2} \left(\frac{1}{3} - \frac{7}{9}\right)^{-1} + 4 = \left(\frac{6}{4} - \frac{3}{4}\right)^{-2} \left(\frac{3}{9} - \frac{7}{9}\right)^{-1} + 4 =$
 $= \left(\frac{3}{4}\right)^{-2} \cdot \left(-\frac{4}{9}\right)^{-1} + 4 = \left(\frac{4}{3}\right)^2 \cdot \left(-\frac{9}{4}\right) + 4 = \frac{-4^2 \cdot 3^2}{3^2 \cdot 4} + 4 = -4 + 4 = 0$

31. a) $\frac{3^6 \cdot 2^5 \cdot 5^2}{9^3 \cdot 4^3 \cdot 5} = \frac{3^6 \cdot 2^5 \cdot 5^2}{3^6 \cdot 2^6 \cdot 5} = \frac{5}{2}$

b) $\frac{3^4 \cdot 16 \cdot 9^{-1}}{5^{-1} \cdot 3^5} = \frac{3^4 \cdot 2^4 \cdot 3^{-2}}{5^{-1} \cdot 3^5} = \frac{2^4 \cdot 5}{3^2 \cdot 3} = \frac{16 \cdot 5}{27} = \frac{80}{27}$

c) $\frac{15^2 \cdot 8^{-1}}{6^3 \cdot 10^2} = \frac{(5 \cdot 3)^2 \cdot 2^{-3}}{(3 \cdot 2)^3 \cdot (5 \cdot 2)^2} = \frac{5^2 \cdot 3^2 \cdot 2^{-3}}{3^3 \cdot 2^3 \cdot 5^2 \cdot 2^2} = \frac{1}{3 \cdot 2^8} = \frac{1}{3 \cdot 256} = \frac{1}{768}$

d) $\frac{a^{-3} b^{-4} c^7}{a^{-5} b^2 c^{-1}} = \frac{a^5 c^7 \cdot c}{a^3 \cdot b^4 \cdot b^2} = \frac{a^2 \cdot c^8}{b^6}$

32. a) $\sqrt[5]{a^2} \cdot \sqrt{a} = a^{\frac{2}{5}} \cdot a^{\frac{1}{2}} = a^{\frac{4}{10}} \cdot a^{\frac{5}{10}} = a^{\frac{9}{10}} = \sqrt[10]{a^9}$

b) $\frac{\sqrt[3]{x^2}}{\sqrt{x}} = \frac{x^{\frac{2}{3}}}{x^{\frac{1}{2}}} = \frac{x^{\frac{4}{6}}}{x^{\frac{3}{6}}} = x^{\frac{4}{6} - \frac{3}{6}} = x^{\frac{1}{6}} = \sqrt[6]{x}$

c) $\frac{1}{\sqrt[4]{a^3}} = \frac{1}{a^{\frac{3}{4}}} = a^{-\frac{3}{4}} = \sqrt[4]{a^{-3}}$

33. a) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$ b) $\sqrt[3]{343} = \sqrt[3]{7^3} = 7$ c) $\sqrt[4]{625} = \sqrt[4]{5^4} = 5$

d) $\sqrt{0'25} = \sqrt{0'5^2} = 0'5$ e) $\sqrt[3]{8^4} = \sqrt[3]{2^{12}} = 2^{\frac{12}{3}} = 2^4 = 16$

f) $\sqrt[3]{0'001} = \sqrt[3]{0'1^3} = 0'1$

$$34. \quad a) \frac{1}{\sqrt{2}} = \frac{1}{2^{1/2}} = 2^{-1/2} \quad b) (-32)^{1/5} = (-2^5)^{1/5} = (-2)^{5/5} = -2$$

$$c) (\sqrt[8]{2})^4 = 2^{4/8} = 2^{1/2}$$

$$35. \quad a) 4 \cdot \frac{1}{3} \cdot \left(-\frac{3}{2}\right)^3 = 2^2 \cdot \frac{1}{3} \cdot \frac{3^3 \cdot (-1)^3}{2^3} = -\frac{3^2}{2} = -\frac{9}{2}$$

$$b) \left(\frac{2}{a}\right)^{-1} \cdot \frac{1}{8} = \left(-\frac{1}{2}\right)^4 \cdot \left(\frac{3^2}{2}\right) \cdot \left(\frac{1}{2^3}\right) = \frac{1^4}{2^4} \cdot \frac{3^2}{2} \cdot \frac{1}{2^3} = \frac{3^2}{2^7}$$

$$c) \frac{(-5)^3 \cdot (-8)^3 \cdot (-9)^2}{15^2 \cdot 20^4} = \frac{+5^3 \cdot 2^9 \cdot 3^4}{(5 \cdot 3)^2 \cdot (2^2 \cdot 5)^4} = \frac{5^3 \cdot 2^9 \cdot 3^4}{5^2 \cdot 3^2 \cdot 2^8 \cdot 5^4} = \frac{2 \cdot 3^2}{5^3 \cdot 5} = \frac{2 \cdot 3^2}{5^4} = \frac{2 \cdot 9}{625} = \frac{18}{625}$$

$$d) \frac{(-30)^{-1} \cdot 15^2}{16^3} = \frac{-15^2}{30 \cdot 16^3} = \frac{-(5 \cdot 3)^2}{(5 \cdot 2 \cdot 3) \cdot (2 \cdot 2^3)^3} = \frac{5^2 \cdot 3^2}{5 \cdot 2 \cdot 3 \cdot 2^3 \cdot 5^3} = \frac{5^2 \cdot 3^2}{5^4 \cdot 2^4 \cdot 3} = \frac{5^2 \cdot 3^2}{5^3 \cdot 2^4} = \frac{5 \cdot 9}{160} = \frac{45}{160} = \frac{9}{32}$$

$$36. \quad a) \frac{\sqrt[4]{a^3} \cdot a^{-1}}{a \sqrt{a}} = \frac{a^{3/4} \cdot a^{-1}}{a \cdot a^{1/2}} = \frac{a^{3/4} \cdot a^{-1}}{a^{1/2} \cdot a^{2/4}} = a^{3/4 - 1 - 1/4} = a^{-3/4} = \sqrt[4]{a^{-3}} = \frac{1}{\sqrt[4]{a^3}}$$

$$b) 16^{1/4} \cdot \sqrt[3]{\frac{1}{4}} \cdot \frac{1}{\sqrt[4]{4}} = 2^{4/4} \cdot \frac{1}{2^{2/3}} \cdot \frac{1}{2^{1/2}} = \frac{2}{2^{2/3}} \cdot \frac{1}{2^{1/2}} = \frac{2}{2^{2/3 + 1/2}} = \frac{2}{2^{7/6}} = 2^{1 - 7/6} = 2^{-1/6} = \frac{1}{\sqrt[6]{2}}$$

Radicals

$$37. \quad a) 2\sqrt[3]{3} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{24}$$

$$b) 4\sqrt[3]{\frac{1}{4}} = \sqrt[3]{\frac{4^3}{4}} = \sqrt[3]{4^2} = \sqrt[3]{16}$$

$$c) 2 \frac{2}{x} \sqrt{\frac{3x}{8}} = \sqrt{\frac{2^2 \cdot 3x}{x^4 \cdot 8}} = \sqrt{\frac{3}{2x}}$$

$$d) \frac{3}{5} \sqrt[3]{\frac{25}{9}} = \sqrt[3]{\frac{3^3 \cdot 25}{5^3 \cdot 9}} = \sqrt[3]{\frac{8^3 \cdot 8^2}{5^3 \cdot 3^2}} = \sqrt[3]{\frac{3^7}{5}}$$

$$e) 2\sqrt[4]{4} = \sqrt[4]{2^4 \cdot 4} = \sqrt[4]{2^6} = 2^{6/4} = 2^{3/2} = \sqrt{2^3} = \sqrt{8}$$

$$f) \frac{1}{5} \sqrt[3]{15} = \sqrt[3]{\frac{1 \cdot 15}{5^3}} = \sqrt[3]{\frac{3}{5^2}} = \sqrt[3]{\frac{3}{25}}$$

38. a) $\sqrt[3]{16} = \sqrt[3]{2^4} = \sqrt[3]{2^3 \cdot 2} = 2 \sqrt[3]{2}$

b) $4 \sqrt{8} = 4 \cdot \sqrt{2^3} = 4 \cdot \sqrt{2^2 \cdot 2} = 4 \cdot 2 \sqrt{2} = 8 \sqrt{2}$

c) $\sqrt{1000} = \sqrt{10^3} = \sqrt{10^2 \cdot 10} = 10 \sqrt{10}$

d) $\sqrt[3]{80a^5} = \sqrt[3]{2^3 \cdot a^3} = \sqrt[3]{2^3 \cdot a^3 \cdot a^2} = 2 \cdot a \cdot \sqrt[3]{a^2}$

e) $\sqrt{\frac{125a^2}{16b}} = \sqrt{\frac{5^3 \cdot a^2}{2^4 \cdot b}} = \sqrt{\frac{5^2 \cdot 5 \cdot a^2}{2^2 \cdot 2^2 \cdot b}} = \frac{5a}{2 \cdot 2} \sqrt{\frac{5}{b}} = \frac{5a}{4} \sqrt{\frac{5}{b}}$

f) $\sqrt{\frac{4}{9} + \frac{1}{9}} = \sqrt{\frac{9}{9} + \frac{4}{36}} = \sqrt{\frac{13}{36}} = \sqrt{\frac{13}{6^2}} = \frac{1}{6} \sqrt{13}$

39. a) $\sqrt[3]{24} = \sqrt[3]{2^3 \cdot 3} = 2 \sqrt[3]{3}$

b) $\sqrt[6]{27} = \sqrt[6]{3^3} = 3^{3/6} = 3^{1/2} = \sqrt{3}$

c) $\sqrt[3]{-108} = -\sqrt[3]{2^2 \cdot 3^3} = -3 \sqrt[3]{2^2} = -3 \sqrt[3]{4}$

d) $\sqrt[12]{64y^3} = \sqrt[12]{2^6 \cdot y^3} = 2^{6/12} \cdot y^{3/12} = 2^{1/2} \cdot y^{1/4} = \sqrt{2} \cdot \sqrt[4]{y}$

e) $\sqrt[4]{\frac{81}{64}} = \sqrt[4]{\frac{3^4}{2^6}} = 3 \cdot \frac{1}{2^{3/2}} = 3 \cdot \frac{1}{2^{3/2}} = 3 \cdot \frac{1}{\sqrt{2^3}} = \frac{3\sqrt{2}}{4}$

f) $\sqrt[3]{625} = \sqrt[3]{5^4} = \sqrt[3]{5^3 \cdot 5} = 5 \sqrt[3]{5} = 1$

40. a) $\sqrt[4]{4}, \sqrt[3]{3}, \sqrt{2} \Rightarrow \sqrt[4]{4} = \sqrt{2} < \sqrt[3]{3}$

rationale
el. determinen

$\sqrt[4]{4} = \sqrt[4]{2^2} = \sqrt[2]{2}, \sqrt[3]{3} = \sqrt[3]{3^2}, \sqrt{2} = \sqrt[2]{2^2} = \sqrt[4]{64}$

b) $\sqrt{6}, \sqrt[3]{9} \Rightarrow \sqrt[3]{9} < \sqrt{6}$

$\sqrt{6} = \sqrt[6]{6^3} = \sqrt[6]{2 \cdot 16}, \sqrt[3]{9} = \sqrt[6]{4^3} = \sqrt[6]{16}$

c) $\sqrt[4]{6}, \sqrt[5]{10} \Rightarrow \sqrt[4]{6} < \sqrt[5]{10}$

$\sqrt[4]{6} = \sqrt[20]{6^5} = \sqrt[20]{7776}, \sqrt[5]{10} = \sqrt[20]{10^4} = \sqrt[20]{10000}$

d) $\sqrt[4]{72}, \sqrt[3]{9}, \sqrt[4]{100} \Rightarrow \sqrt[3]{9} < \sqrt[4]{100} < \sqrt[4]{72}$

$\sqrt[4]{72} = \sqrt[4]{3^3 \cdot 2^3} = \sqrt[2]{3 \cdot 3 \cdot 2 \cdot 2} = \sqrt[2]{6 \cdot 5 \cdot 6}, \sqrt[4]{100} = \sqrt[2]{10 \cdot 2} = \sqrt[2]{100}$

Paß 29

41. a) $4 \sqrt{23} \cdot 5 \sqrt{6} = 20 \sqrt{6 \cdot 23} = 20 \cdot \sqrt{3 \cdot 2 \cdot 3^2} = 20 \sqrt{3^2 \cdot 3^2 \cdot 2} = 20 \cdot 3 \cdot 3 \sqrt{2} = 180 \sqrt{2}$

b) $2 \sqrt{\frac{4}{3}} \cdot \sqrt{\frac{27}{8}} = 2 \cdot \sqrt{\frac{2^2 \cdot 3^3}{3 \cdot 2^3}} = 2 \sqrt{\frac{3^2}{2}} = 2 \cdot 3 \cdot \frac{1}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 3 \sqrt{2}$

$$c) \sqrt{2} \cdot \sqrt{\frac{1}{8}} = \sqrt{\frac{2}{2^3}} = \sqrt{\frac{1}{2^2}} = \frac{1}{2}$$

$$d) (\sqrt[3]{12})^2 = (\sqrt[3]{2^2 \cdot 3})^2 = \sqrt[3]{2^4 \cdot 3^2} = 2 \sqrt[3]{2 \cdot 3^2} = 2 \sqrt[3]{18}$$

$$e) (\sqrt[6]{32})^3 = (\sqrt[6]{2^5})^3 = (2^5)^{3/6} = (2^5)^{1/2} = \sqrt{2^5} = \sqrt{2^2 \cdot 2^2 \cdot 2} = 2 \cdot 2 \sqrt{2} = 4\sqrt{2}$$

$$f) \sqrt[3]{24} \cdot \sqrt[3]{3} = \sqrt[3]{\frac{2^3 \cdot 3}{3}} = \sqrt[3]{2^3} = 2$$

$$42. a) \sqrt[4]{\sqrt[3]{4}} = \sqrt[12]{4} = \sqrt[12]{2^2} = 2^{2/12} = 2^{1/6} = \sqrt[6]{2}$$

$$b) \sqrt[3]{2 \sqrt[4]{8}} = \sqrt[3]{\sqrt[4]{2^4 \cdot 2^3}} = \sqrt[12]{2^7} = \sqrt[12]{128}$$

$$c) (\sqrt[4]{a^3} \cdot \sqrt[5]{a^4}) : \sqrt{a} = \frac{\sqrt[20]{a^{15} \cdot a^{16}}}{\sqrt[20]{a^{10}}} = \sqrt[20]{\frac{a^{15} \cdot a^{16}}{a^{10}}} = \sqrt[20]{a^{21}} = a^{20} \sqrt[20]{a}$$

$$43. a) \frac{2\sqrt{3}}{\sqrt{18}} = \frac{2\sqrt{3}}{\sqrt{2 \cdot 3^2}} = \frac{2\sqrt{3}}{3\sqrt{2}} = \frac{2\sqrt{3}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{6}}{3 \cdot 2} = \frac{\sqrt{6}}{3}$$

$$b) \frac{2}{\sqrt[3]{2}} = \frac{2}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{2\sqrt[3]{2^2}}{2} = \sqrt[3]{2^2} = \sqrt[3]{4}$$

$$c) \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{(\sqrt{2}-1)\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$d) \frac{3}{3+\sqrt{3}} = \frac{3}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{3(3-\sqrt{3})}{9-3} = \frac{3(3-\sqrt{3})}{6} = \frac{3-\sqrt{3}}{2}$$

$$e) \frac{\sqrt{72} + 3\sqrt{32} - \sqrt{8}}{\sqrt{8}} = \frac{\sqrt{2^3 \cdot 3^2} + 3 \cdot \sqrt{2^5} - \sqrt{2^3}}{\sqrt{2^3}} =$$

$$= \frac{2 \cdot 3 \cdot \sqrt{2} + 3 \cdot 2 \cdot 2\sqrt{2} - 2\sqrt{2}}{2\sqrt{2}} = \frac{3\sqrt{2} + 6\sqrt{2} - \sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} =$$

$$= \frac{8\sqrt{2}\sqrt{2}}{2} = 8$$

$$44. a) 5\sqrt{125} + 6\sqrt{45} - 7\sqrt{20} + \frac{3}{2}\sqrt{80} = 5 \cdot \sqrt{5^3} + 6 \cdot \sqrt{3^2 \cdot 5} - 7\sqrt{2^2 \cdot 5} + \frac{3}{2} \sqrt{2^4 \cdot 5}$$

$$= 5 \cdot 5\sqrt{5} + 6 \cdot 3\sqrt{5} - 7 \cdot 2\sqrt{5} + \frac{3}{2} \cdot 4\sqrt{5} = 25\sqrt{5} + 18\sqrt{5} - 14\sqrt{5} + 6\sqrt{5} =$$

$$= 35\sqrt{5}$$

$$b) \sqrt[3]{16} + 2\sqrt[3]{2} - \sqrt[3]{54} - \frac{21}{5}\sqrt[3]{250} = \sqrt[3]{2^4} + 2\sqrt[3]{2} - \sqrt[3]{3^3 \cdot 2} - \frac{21}{5}\sqrt[3]{5^3 \cdot 2} =$$

$$= 2\sqrt[3]{2} + 2\sqrt[3]{2} - 3\sqrt[3]{2} - \frac{21}{5} \cdot 5\sqrt[3]{2} = -20\sqrt[3]{2}$$

$$c) \sqrt{125} + \sqrt{54} - \sqrt{45} - \sqrt{24} = \sqrt{5^3} + \sqrt{3^3 \cdot 2} - \sqrt{3^2 \cdot 5} - \sqrt{2^3 \cdot 3}$$

$$= 5\sqrt{5} + 3\sqrt{6} - 3\sqrt{5} - 2\sqrt{6} = 2\sqrt{5} + \sqrt{6}$$

(cont. 44)

$$\begin{aligned} d) (\sqrt{2} + \sqrt{3})(\sqrt{6} - 1) &= \sqrt{2} \cdot \sqrt{6} + \sqrt{2} \cdot (-1) + \sqrt{3} \cdot \sqrt{6} + \sqrt{3} \cdot (-1) = \\ &= \sqrt{2^2 \cdot 3} - \sqrt{2} + \sqrt{3^2 \cdot 2} + \sqrt{3} = 2\sqrt{3} - \sqrt{2} + 3\sqrt{2} - \sqrt{3} \\ &= \sqrt{3} + 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} 45. a) 3\sqrt[3]{16} - 2\sqrt[3]{250} + 5\sqrt[3]{54} - 4\sqrt[3]{2} &= 3\sqrt[3]{2^4} - 2\sqrt[3]{2 \cdot 5^3} + 5\sqrt[3]{2 \cdot 3^3} - 4\sqrt[3]{2} = \\ &= 3 \cdot 2\sqrt[3]{2} - 2 \cdot 5\sqrt[3]{2} + 5 \cdot 3\sqrt[3]{2} - 4\sqrt[3]{2} = 6\sqrt[3]{2} - 10\sqrt[3]{2} + 15\sqrt[3]{2} - 4\sqrt[3]{2} = \\ &= 7\sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} b) \sqrt{\frac{2}{5}} - 4\sqrt{\frac{18}{125}} + \frac{1}{3}\sqrt{\frac{8}{45}} &= \sqrt{\frac{2}{5}} - 4 \cdot \sqrt{\frac{3^2 \cdot 2}{5^3}} + \frac{1}{3}\sqrt{\frac{2^3}{3^2 \cdot 5}} = \\ &= \sqrt{\frac{2}{5}} - 4 \cdot \frac{3}{5}\sqrt{\frac{2}{5}} + \frac{2}{3 \cdot 3}\sqrt{\frac{2}{5}} = \sqrt{\frac{2}{5}} - \frac{12}{5}\sqrt{\frac{2}{5}} + \frac{2}{9}\sqrt{\frac{2}{5}} = \\ &= \left(1 - \frac{12}{5} + \frac{2}{9}\right)\sqrt{\frac{2}{5}} = \frac{45 - 108 + 10}{45}\sqrt{\frac{2}{5}} = -\frac{53}{45}\sqrt{\frac{2}{5}} \end{aligned}$$

$$\begin{aligned} c) 7\sqrt[3]{81a^4} - 2\sqrt[3]{3a^4} + \frac{\sqrt[3]{3a}}{5} &= 7\sqrt[3]{3^4 a^4} - 2\sqrt[3]{3a^4} + \frac{1}{5}\sqrt[3]{3a} = \\ &= 7 \cdot 3\sqrt[3]{3a^4} - 2a\sqrt[3]{3a} + \frac{1}{5}\sqrt[3]{3a} = 21\sqrt[3]{3a^4} - 2a\sqrt[3]{3a} + \frac{1}{5}\sqrt[3]{3a} = \\ &= \left(21 - 2a + \frac{1}{5}\right)\sqrt[3]{3a} = \frac{105 - 2a + 1}{5}\sqrt[3]{3a} = \\ &= \left(\frac{106 - 2a}{5}\right)\sqrt[3]{3a} \end{aligned}$$

$$\begin{aligned} 46. a) (\sqrt{3} + \sqrt{2})^2 - (\sqrt{3} - \sqrt{2})^2 &= 3 + 2\sqrt{6} + 2 - (3 - 2\sqrt{6} + 2) = \\ &= 5 + 2\sqrt{6} - (5 - 2\sqrt{6}) = 5 + 2\sqrt{6} - 5 + 2\sqrt{6} = 4\sqrt{6} \end{aligned}$$

$$b) (\sqrt{6} + \sqrt{5})2\sqrt{2} = 2\sqrt{2^2 \cdot 3} + 2\sqrt{5 \cdot 2} = 4\sqrt{3} + 2\sqrt{10}$$

$$c) (\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{6}) = (\sqrt{5})^2 - (\sqrt{6})^2 = 5 - 6 = -1$$

$$\begin{aligned} d) (2\sqrt{5} - 3\sqrt{2})^2 &= (2\sqrt{5})^2 - 2 \cdot (2\sqrt{5})(3\sqrt{2}) + (3\sqrt{2})^2 = \\ &= 4 \cdot 5 - 12\sqrt{10} + 9 \cdot 2 = 20 - 12\sqrt{10} + 18 = 38 - 12\sqrt{10} \end{aligned}$$

$$e) (\sqrt{2} - 1)(\sqrt{2} + 1)\sqrt{3} = [(\sqrt{2})^2 - 1^2]\sqrt{3} = (2 - 1)\sqrt{3} = \sqrt{3}$$

47. a) $\frac{2\sqrt{3}-\sqrt{2}}{\sqrt{18}} = \frac{2\sqrt{3}-\sqrt{2}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{(2\sqrt{3}-\sqrt{2})\sqrt{2}}{3 \cdot 2} = \frac{2\sqrt{6}-2}{6} = \frac{\sqrt{6}-1}{3}$

b) $\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{12}} = \frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(2\sqrt{3}+\sqrt{2})\sqrt{3}}{2 \cdot 3} = \frac{6+\sqrt{6}}{6} = 1+\frac{\sqrt{6}}{6}$

c) $\frac{1}{2(\sqrt{3}-\sqrt{5})} = \frac{1}{2(\sqrt{3}-\sqrt{5})} \cdot \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{\sqrt{3}+\sqrt{5}}{2(3-5)} = -\frac{\sqrt{3}+\sqrt{5}}{4}$

d) $\frac{3}{\sqrt{5}-2} = \frac{3}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{3(\sqrt{5}+2)}{5-4} = 3\sqrt{5}+6$

e) $\frac{11}{2\sqrt{5}+3} = \frac{11}{2\sqrt{5}+3} \cdot \frac{2\sqrt{5}-3}{2\sqrt{5}-3} = \frac{11(2\sqrt{5}-3)}{4 \cdot 5-9} = \frac{11(2\sqrt{5}-3)}{20-9} = \frac{11(2\sqrt{5}-3)}{11}$
 $= 2\sqrt{5}-3$

f) $\frac{3\sqrt{6}+2\sqrt{2}}{3\sqrt{3}+2} = \frac{3\sqrt{6}+2\sqrt{2}}{3\sqrt{3}+2} \cdot \frac{3\sqrt{3}-2}{3\sqrt{3}-2} = \frac{(3\sqrt{6}+2\sqrt{2})(3\sqrt{3}-2)}{9 \cdot 3-4} =$
 $= \frac{9\sqrt{18}+2 \cdot 6\sqrt{6}-6\sqrt{6}-4\sqrt{2}}{23} = \frac{27\sqrt{2}-4\sqrt{2}}{23} = \frac{23\sqrt{2}}{23} = \sqrt{2}$

Numbers approximations

48. a) $\epsilon_a = 500 \text{ €}$ $\epsilon_n < 500 / 122000 < 3,9 \cdot 10^{-3}$

b) $\epsilon_a = 500.000 \text{ €}$ $\epsilon_n < 500.000 / 25.000.000 < 0,02$

c) $\epsilon_a = 50 \text{ €}$ $\epsilon_n < 50 / 642.506 < 7,7 \cdot 10^{-5}$

d) $\epsilon_a = 50 \text{ €}$ $\epsilon_n < 50 / 3200 < 0,016$

49. a) ~~840.000~~ ^{843.754} ~~cents~~ (approximatió)

$\epsilon_a = 843.754 - 840.000 = 3754$

$\epsilon_n < 3754 / 843.754 < 0,004$

b) 29.000 €

$\epsilon_a = 29.000 - 28.782 = 218$

$\epsilon_n = 218 / 28.782 \approx 7,5 \cdot 10^{-3}$

c) 0,04 segons

$\epsilon_a = 0,04 - 0,0375 = 0,0025$

$\epsilon_n < 0,0025 / 0,0375 < 0,06$

d) 49 milions

$\epsilon_a = 49.000.000 - 48.759.450 = 240.550$

$\epsilon_n \approx 240.550 / 48.759.450 \approx 4,9 \cdot 10^{-3}$

50. a) $8 \cdot 10^5$ $E_a = 5 \cdot 10^4$

$E_n < 5 \cdot 10^4 / 8 \cdot 10^5 < 0'0625$

b) $5'23 \cdot 10^6$ $E_a = 5 \cdot 10^3$

$E_n < 5 \cdot 10^3 / 5'23 \cdot 10^6 < 0'00096$

c) $1'372 \cdot 10^7$ $E_a = 5 \cdot 10^3$

$E_n < 5 \cdot 10^3 / 1'372 \cdot 10^7 < 0'00036$

d) $2'5 \cdot 10^{-4}$ $E_a = 5 \cdot 10^{-6}$

$E_n < 5 \cdot 10^{-6} / 2'5 \cdot 10^{-4} < 0'2$

e) $1'7 \cdot 10^{-6}$ $E_a = 5 \cdot 10^8$

$E_n < 5 \cdot 10^8 / 1'7 \cdot 10^{-6} < 0'029$

f) $4 \cdot 10^{-5}$ $E_a = 5 \cdot 10^6$

$E_n < 5 \cdot 10^{-6} / 4 \cdot 10^{-5} < 0'125$

Notació científica

51. a) $\frac{2'8 \cdot 10^{-5}}{6'2 \cdot 10^{-12}} = 0'452 \cdot 10^7 = 4'52 \cdot 10^6$

b) $\frac{(7'2 \cdot 10^{-4})^3}{5'3 \cdot 10^{-9}} = \frac{7'2^3 \cdot 10^{-18}}{5'3 \cdot 10^{-9}} = 70'4 \cdot 10^{-9} = 7'04 \cdot 10^{-8}$

c) $7'86 \cdot 10^5 \cdot 1'4 \cdot 10^6 + 5'2 \cdot 10^9 =$

$= 7'86 \cdot 10^9 + 1'40 \cdot 10^9 + 5'2 \cdot 10^9 =$

$= -56'2 \cdot 10^9 = -5'62 \cdot 10^5$

d) $\frac{(3 \cdot 10^{-10} + 7 \cdot 10^9)}{(7 \cdot 10^8 - 5 \cdot 10^5)} = \frac{0'3 \cdot 10^{-9} + 7 \cdot 10^{-9}}{70 \cdot 10^5 - 5 \cdot 10^5} = \frac{7'3 \cdot 10^{-9}}{65 \cdot 10^5} =$

$= 0'112 \cdot 10^{-14} = 1'12 \cdot 10^{-15}$

Pàg 30

52. $\frac{2 \cdot 10^7 - 3 \cdot 10^{-5}}{4 \cdot 10^6 + 10^5} = \frac{0'02 \cdot 10^{-5} - 3 \cdot 10^{-5}}{40 \cdot 10^5 + 10^5} = \frac{-2'98 \cdot 10^{-5}}{41 \cdot 10^5} =$

$= -0'0727 \cdot 10^{-10} = -7'27 \cdot 10^{-12}$

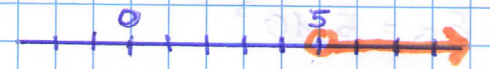
$(-0'07268 \cdot 10^{-10} = -7'268 \cdot 10^{-12})$

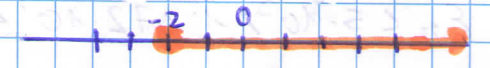
$$53. \frac{60 \cdot 000^3 \cdot 0'00002^4}{100^2 \cdot 72 \cdot 000 \cdot 000 \cdot 0'00025} = \frac{(8 \cdot 10^4)^3 \cdot (2 \cdot 10^5)^4}{10^4 \cdot 7'2 \cdot 10^7 \cdot (2 \cdot 10^{-4})^5}$$

$$= \frac{6^3 \cdot 10^{12} \cdot 2^4 \cdot 10^{20}}{10^4 \cdot 7'2 \cdot 10^7 \cdot 2^5 \cdot 10^{-20}} = \frac{6^3 \cdot 10^{12}}{7'2 \cdot 2 \cdot 10^{11}} = 15 \cdot 10 = 150$$

Interval i valor absolut.

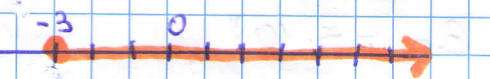
54. a) $-3 \leq x \leq 2$  $[-3, 2]$

b) $5 < x$  $(5, +\infty)$

c) $x \geq -2$  $[-2, +\infty)$

d) $-2 \leq x < 3/2$  $[-2, 3/2)$

e) $4 < x < 4'1$  $(4, 4'1)$

f) $-3 \leq x$  $[-3, +\infty)$


55. a) $[-2, 7]$ $-2 \leq x \leq 7$ b) $[13, +\infty)$ $13 \leq x$ c) $(-\infty, 0)$ $x < 0$


d) $(-3, 0]$ $-3 < x \leq 0$ e) $[\frac{3}{2}, 6)$ $\frac{3}{2} \leq x < 6$ f) $(-\infty, +\infty)$ $-\infty < x < +\infty$

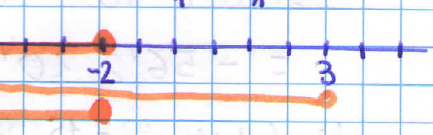
56. a) $A = [-3, 2]$, $B = [0, 5]$ $A \cap B = [0, 2]$

b) $I = [2, +\infty)$, $J = (0, 10)$ $I \cap J = [2, 10)$

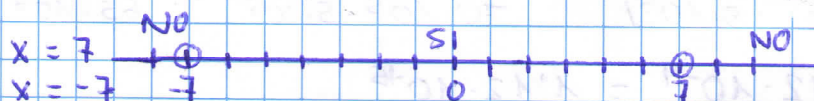
57. a) $x < 3$ o $x \geq 5$ $(-\infty, 3) \cup [5, +\infty)$ 

b) $x > 0$ i $x < 4$ $(0, 4)$ 

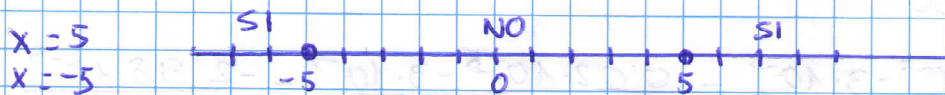
c) $x \leq -1$ o $x > 1$ $(-\infty, -1] \cup (1, +\infty)$ 

d) $x < 3$ i $x \leq -2$ $(-\infty, -2] \cup (-\infty, 3)$ 

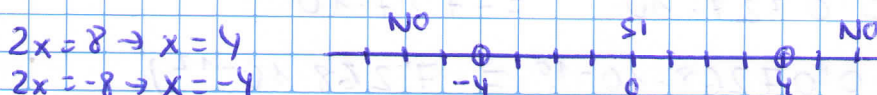
58. a) $|x| < 7$ $(-7, 7)$



b) $|x| \geq 5$ $(-\infty, -5] \cup [5, +\infty)$

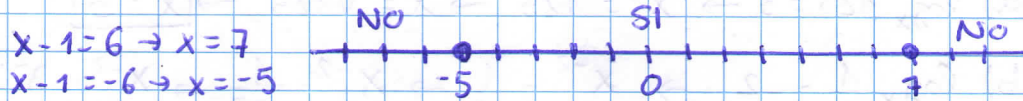


c) $|2x| < 8$ $(-4, 4)$

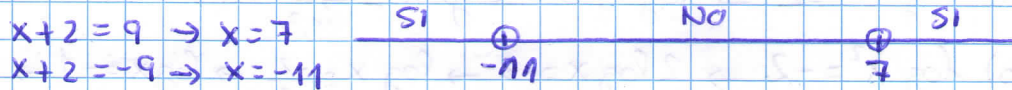


(cont. 58)

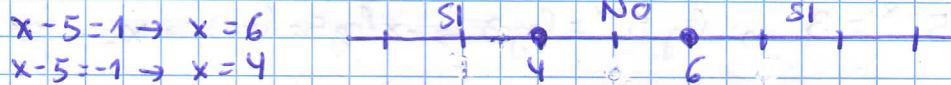
d) $|x-1| \leq 6$ $[-5, 7]$



e) $|x+2| > 9$ $(-\infty, -11) \cup (7, +\infty)$



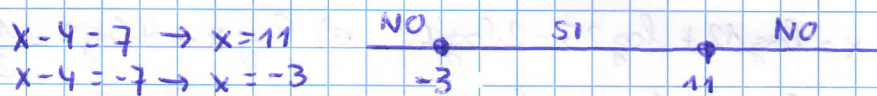
f) $|x-5| \geq 1$ $(-\infty, 4] \cup [6, +\infty)$



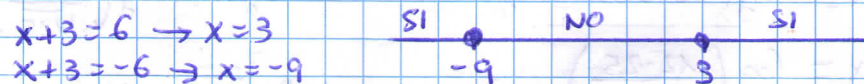
59. a) $|x-2|=5$ $x=7$ & $x=-3$

$x-2=5 \rightarrow x=7$
 $x-2=-5 \rightarrow x=-3$

b) $|x-4| \leq 7$ $[-3, 11]$ $-3 \leq x \leq 11$



c) $|x+3| \geq 6$ $(-\infty, -9] \cup [3, +\infty)$ $x \leq -9$ & $x \geq 3$



60. $d(a, b) = |a - b|$

a) $7; 3 \rightarrow d(7, 3) = |7 - 3| = 4$

b) $5; 11 \rightarrow d(5, 11) = |5 - 11| = |-6| = 6$

c) $-3; -9 \rightarrow d(-3, -9) = |-3 - (-9)| = |-3 + 9| = 6$

d) $-3; 4 \rightarrow d(-3, 4) = |-3 - 4| = |-7| = 7$

Logarithmes

61. a) $\log_2 64 + \log_2 \frac{1}{4} - \log_3 9 - \log_2 \sqrt{2} =$

$= \log_2 2^6 + \log_2 2^{-2} - \log_3 3^2 - \log_2 2^{1/2} =$

$= 6 \log_2 2 + (-2) \log_2 2 - 2 \log_3 3 - \frac{1}{2} \log_2 2 = 6 - 2 - 2 - \frac{1}{2} =$

$= 2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

$$b) \log_2 \frac{1}{32} + \log_3 \frac{1}{27} - \log_2 1 = \log_2 2^{-5} + \log_3 3^{-3} - \log_2 2^0 = -5 \log_2 2 + (-3) \log_3 3 - 0 \cdot \log_2 2 = -5 - 3 - 0 = -8$$

62. a) $\log_x 125 = 3 \rightarrow x^3 = 125 \rightarrow x = \sqrt[3]{125} \rightarrow \boxed{x=5}$

b) $\log_x \frac{1}{9} = -2 \rightarrow x^{-2} = \frac{1}{9} \rightarrow x^{-2} = \frac{1}{3^2} \rightarrow x^{-2} = 3^{-2} \rightarrow \boxed{x=3}$

63. a) $\log 3^x = 2 \rightarrow x \log 3 = 2 \rightarrow x = \frac{2}{\log 3} \rightarrow x \approx 4,19$

b) $\log x^2 = -2 \rightarrow 2 \log x = -2 \rightarrow \log x = -\frac{2}{2} \rightarrow \log x = -1 \rightarrow x = 10^{-1} = \frac{1}{10}$

c) $7^x = 115 \rightarrow \log 7^x = \log 115 \rightarrow x \log 7 = \log 115 \rightarrow x = \frac{\log 115}{\log 7} \approx 2,44$

d) $5^{-x} = 3 \rightarrow \log 5^{-x} = \log 3 \rightarrow -x \log 5 = \log 3 \rightarrow x = -\frac{\log 3}{\log 5} \rightarrow x \approx -0,68$

65. a) $\log x = \log 17 + \log 13$ b) $\ln x = \ln 36 - \ln 9$ c) $\ln x = 3 \ln 5$

$$\log x = \log (17 \cdot 13) \quad \ln x = \ln (36/9) \quad \ln x = \ln 5^3$$

$$x = 17 \cdot 13$$

$$x = 36/9$$

$$x = 5^3$$

$$\boxed{x = 221}$$

$$\boxed{x = 4}$$

$$\boxed{x = 125}$$

d) $\log x = \log 12 + \log 25 - 2 \log 6$ e) $\log x = 4 \log 2 - \frac{1}{2} \log 25$

$$\log x = \log 12 + \log 25 - \log 6^2 \quad \log x = \log 2^4 - \log 25^{1/2}$$

$$\log x = \log (12 \cdot 25) - \log 6^2 \quad \log x = \log \left(\frac{2^4}{\sqrt{25}} \right)$$

$$\log x = \log \left(\frac{12 \cdot 25}{6^2} \right)$$

$$\boxed{x = \frac{16}{5}}$$

$$x = \frac{12 \cdot 25}{6^2} = \left(\frac{3 \cdot 2^2 \cdot 25}{3^2 \cdot 2^2} \right)$$

$$\boxed{x = \frac{25}{3}}$$

64. a) $\log \sqrt{148} \approx 1,085$ b) $\ln 2^{13} \cdot 10^{11} \approx 26,161$ c) $\log 7^{12} \cdot 10^{-5} = -4,143$

d) $\log_3 4219 = \frac{\log 4219}{\log 3} \approx 3,42$ e) $\log_5 1,95 = \frac{\log 1,95}{\log 5} \approx 0,415$

f) $\log_2 0,034 = \frac{\log 0,034}{\log 2} \approx -4,878$

66. $\log 3 = 0,477$

$$\log 30 = \log (3 \cdot 10) = \log 3 + \log 10 = 0,477 + 1 = 1,477$$

$$\log 300 = \log (3 \cdot 100) = \log 3 + \log 100 = \log 3 + \log 10^2 = \log 3 + 2 \log 10 = 0,477 + 2$$

$$2,477$$

(cont 66)

$$\log 3000 = \log(3 \cdot 10^3) = \log 3 + \log 10^3 = \log 3 + 3 \log 10 = 0.477 + 3 = 3.477$$

$$\log 0.3 = \log(3 \cdot 10^{-1}) = \log 3 + \log 10^{-1} = \log 3 - \log 10 = 0.477 - 1 = -0.523$$

$$\log 0.03 = \log(3 \cdot 10^{-2}) = \log 3 + \log 10^{-2} = \log 3 - 2 \log 10 = 0.477 - 2 = -1.523$$

$$\log 0.003 = \log(3 \cdot 10^{-3}) = \log 3 + \log 10^{-3} = \log 3 - 3 \log 10 = 0.477 - 3 = -2.523$$

67. $\log k = 1.4$

a) $\log \frac{k}{100} = \log k - \log 10^2 = 1.4 - 2 = -0.6$

b) $\log(0.1k^2) = \log 10^{-1} + \log k^2 = \log 10 + 2 \log k = -1 + 2 \cdot 1.4 = 1.8$

c) $\log \sqrt[3]{\frac{1}{k}} = \frac{1}{3} (\log \frac{1}{k}) = \frac{1}{3} (\log 1 - \log k) = -\frac{1}{3} \cdot 1.4 = -0.467$

d) $(\log k)^{1/2} = \sqrt{1.4} \approx 1.183$

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68. a) $\log_x \frac{1}{4} = 2$ b) $\log_x 2 = \frac{1}{2}$ c) $\log_x 0.04 = -2$ d) $\log_x 4 = -\frac{1}{2}$

$$x^2 = \frac{1}{4}$$

$$x^{1/2} = 2$$

$$x^{-2} = 0.04$$

$$x^{-1/2} = 4$$

$$x^2 = \left(\frac{1}{2}\right)^2$$

$$x = 2^2$$

$$x^{-2} = 2^2 \cdot 10^{-2}$$

$$\frac{1}{x^{1/2}} = 4$$

$$\boxed{x = \frac{1}{2}}$$

$$\boxed{x = 4}$$

$$x^{-2} = \left(\frac{10}{2}\right)^{-2}$$

$$\frac{1}{4} = x^{1/2}$$

$$\boxed{x = 5}$$

$$\boxed{x = \frac{1}{16}}$$

69. a) $3^x = 0.005$ b) $0.8^x = 17$ c) $e^x = 18$ d) $1.5^x = 15$

$$\log 3^x = \log 0.005$$

$$\log 0.8^x = \log 17$$

$$\ln e^x = \ln 18$$

$$\log 1.5^x = \log 15$$

$$x \log 3 = \log 0.005$$

$$x \log 0.8 = \log 17$$

$$x \ln e = \ln 18$$

$$x \log 1.5 = \log 15$$

$$x = \frac{\log 0.005}{\log 3}$$

$$x = \frac{\log 17}{\log 0.8}$$

$$x = \ln 18$$

$$x = \frac{\log 15}{\log 1.5}$$

$$x = -4.823$$

$$x = -12.697$$

$$x = 2.890$$

$$x = 6.679$$

e) $0.5^x = 0.004$

f) $e^x = 0.1$

$$\log 0.5^x = \log 0.004$$

$$\ln e^x = \ln 0.1$$

$$x \log 0.5 = \log 0.004$$

$$x \ln e = \ln 0.1$$

$$x = \frac{\log 0.004}{\log 0.5}$$

$$x = \ln 0.1$$

$$x = -7.966$$

$$x = -2.303$$

70. a) $x^{2.7} = 19$
 $2.7 \log x = \log 19$
 $\log x = \frac{\log 19}{2.7}$

$\log x = 0.474$
 $x = 10^{0.474}$

$x \approx 2.978$

b) $\log_7 3x = 0.5$
 $\frac{\log 3x}{\log 7} = 0.5$

$\log 3x = 0.5 \log 7$
 $\log 3x = \log 7^{1/2}$

$3x = \sqrt{7}$
 $x = \frac{\sqrt{7}}{3} \approx 0.882$

c) $3^{2+x} = 172$
 $(2+x) \log 3 = \log 172$
 $2 \log 3 + x \log 3 = \log 172$

$x \log 3 = \log 172 - 2 \log 3$
 $x = \frac{\log 172 - 2 \log 3}{\log 3}$

$x = 2.685$

71. $\log k = x$

a) $\log k^2 = 2 \log k = 2x$

b) $\log \frac{k}{100} = \log k - \log 10^2 = \log k - 2 \log 10 = x - 2$

c) $\log \sqrt{10k} = \log (10k)^{\frac{1}{2}} = \frac{1}{2} \log 10 \cdot k = \frac{1}{2} (\log 10 + \log k) = \frac{1+x}{2}$

Problemas aritmètics

72. $\frac{17}{4} + \frac{17}{5} + \frac{17}{8} = 20.000$ ($4+5+8=17$)

① $\frac{17}{4} \text{ --- } x$

$\frac{17}{4} + \frac{17}{5} + \frac{17}{8} = \frac{170}{40} + \frac{136}{40} + \frac{85}{40} = \frac{391}{40}$

② $\frac{17}{5} \text{ --- } y$

① $x = \frac{17 \cdot 20.000}{\frac{391}{40}} = 8.695'65 \text{ € per al primer}$

③ $\frac{17}{8} \text{ --- } z$

② $y = \frac{17 \cdot 20.000}{\frac{391}{40}} = 6.956'52 \text{ € per al segon}$

③ $z = \frac{17 \cdot 20.000}{\frac{391}{40}} = 4.347'83 \text{ € per al tercer}$

73.

Consumeix	recorregut	
litres	Km	

6'4

100

$x = \frac{52 \cdot 100}{6.4} = 812'5 \text{ Km}$

52

x

L'automòbil podria recórrer 812'5 Km amb 52 l.