Pàg 131

① $y=x^2-5$ contínua en tot $\mathbb{R}$

Perquè està definida en tot $\mathbb{R}$.

② $y=\sqrt{5-x}$ contínua en $(-\infty,5]$.

Perquè el seu domini és $(-\infty,5]$

③ a) $y=\dfrac{x+2}{x-3}$ Branca infinita en $x=3$ (asímptota vertical)

$x-3=0 \rightarrow x=3$

b) $y=\dfrac{x^2-3x}{x}$ Discontinuïtat evitable en $x=0$ (li falta aquest punt)

$x=0$

c) $y=\dfrac{x^2-3}{x}$ Branca infinita en $x=0$ (asímptota vertical)

$x=0$

d) $y=\dfrac{1}{x^2}$ Branca infinita en $x=0$ (asímptota vertical)

$x^2=0 \rightarrow x=0$

e) $\begin{cases} 3x-4, & x<3 \\ x+1, & x\geq 3 \end{cases}$ Salt en $x=3$

$3\cdot 3-4=9-4=5$

$3+1=4$

f) $\begin{cases} 3 & \text{si } x\neq 4 \\ 1 & \text{si } x=4 \end{cases}$ Salt en $x=4$

Pàg 134

④ a) $\lim\limits_{x\to 0} \dfrac{3}{x-2} = \dfrac{3}{0-2} = \dfrac{-3}{2}$ b) $\lim\limits_{x\to 0} (\cos x-1) = \cos 0-1 = 1-1=0$

⑤ a) $\lim\limits_{x\to 2} \sqrt{x^2-3x+5} = \sqrt{2^2-3\cdot 2+5} = \sqrt{4-6+5} = \sqrt{3}$

b) $\lim\limits_{x\to 0'1} \log_{10} x = \log_{10} 0'1 = \log_{10} 10^{-1} = -1$

Pàg 135

⑥ $f(x) = \begin{cases} x^3-2x+k & , x \neq 3 \\ 7 & , x=3 \end{cases}$

$$\left.\begin{array}{l} \lim_{x\to 3} (x^3-2x+k) = 3^3-2\cdot 3+k = 27-6+k = 21+k \\ \\ f(3)=7 \end{array}\right\} 21+k=7 \to k=7-21 \quad \boxed{k=-14}$$

Pàg 137

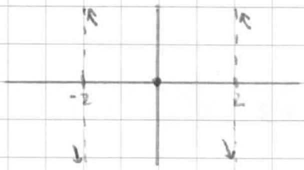
⑦ a) $f(x) = \dfrac{x^3}{x^2-4}$ en $-2, 0, 2$

$$\left.\begin{array}{l} \lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \dfrac{x^3}{x^2-4} = \dfrac{-8}{0^+} = -\infty \\ \\ \lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \dfrac{x^3}{x^2-4} = \dfrac{-8}{0^-} = +\infty \end{array}\right\} \text{No existeix } \lim_{x\to -2} f(x)$$

No cal pq està definida en el punt

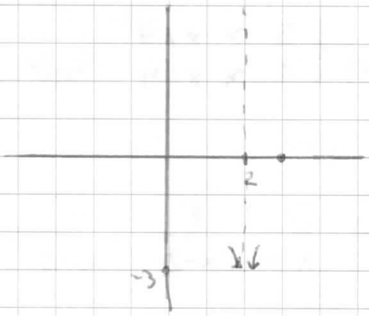
$$\left.\begin{array}{l} \lim_{x\to 0^-} f(x) = \lim_{x\to 0} \dfrac{x^3}{x^2-4} = \dfrac{0}{0^2-4} = \dfrac{0}{-4} = 0 \\ \\ \lim_{x\to 0^+} f(x) = \lim_{x\to 0} \dfrac{x^3}{x^2-4} = \dfrac{0}{-4} = 0 \end{array}\right\} \lim_{x\to 0} f(x) = 0$$

$$\left.\begin{array}{l} \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{x^3}{x^2-4} = \dfrac{2^3}{0^-} = -\infty \\ \\ \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \dfrac{x^3}{x^2-4} = \dfrac{8}{0^+} = +\infty \end{array}\right\} \text{No existeix } \lim_{x\to 2} f(x)$$



b) $f(x) = \dfrac{4x-12}{(x-2)^2}$ en 2, 0 i 3

$$\left.\begin{array}{l} \lim_{x\to 2^-} \dfrac{4x-12}{(x-2)^2} = \dfrac{-4}{0^+} = -\infty \\ \\ \lim_{x\to 2^+} \dfrac{4x-12}{(x-2)^2} = \dfrac{-4}{0^+} = -\infty \end{array}\right\} \lim_{x\to 2} f(x) = -\infty$$



$$\left.\begin{array}{l} \lim_{x\to 0^-} \dfrac{4x-12}{(x-2)^2} = \dfrac{-12}{4} = -3 \\ \\ \lim_{x\to 0^+} \dfrac{4x-12}{(x-2)^2} = \dfrac{-12}{4} = -3 \end{array}\right\} \lim_{x\to 0} f(x) = -3$$

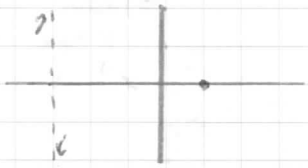
$$\left.\begin{array}{l} \lim_{x\to 3^-} \dfrac{4x-12}{(x-2)^2} = \dfrac{0}{1} = 0 \\ \lim_{x\to 3^+} \dfrac{4x-12}{(x-2)^2} = \dfrac{0}{1} = 0 \end{array}\right\} \lim_{x\to 3} f(x) = 0$$

c) $f(x) = \dfrac{x^2-2x+1}{x^2+2x-3}$ en 1 i -3

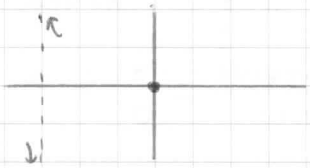
$$\lim_{x\to 1} \frac{x^2-2x+1}{x^2+2x-3} = \left(\frac{0}{0} \text{ INDET}\right) = \lim_{x\to 1} \frac{(x-1)^2}{(x+3)(x-1)} = \lim_{x\to 1} \frac{x-1}{x+3} = \frac{0}{4} = 0$$

$$\begin{array}{r|l} x^2+2x-3 & x-1 \\ \cline{2-2} -x^2+x & x+3 \\ \cline{1-1} 3x-3 & \\ -3x+3 & \\ \cline{1-1} 0 & \end{array}$$

$$\left.\begin{array}{l} \lim\limits_{x\to -3^-} \dfrac{x^2-2x+1}{x^2+2x-3} = \dfrac{9+6+1}{9-6-3} = \dfrac{16}{0^+} = +\infty \\ \lim\limits_{x\to -3^+} \dfrac{x^2-2x+1}{x^2+2x-3} = \dfrac{16}{0^-} = -\infty \end{array}\right\} \text{No existeix } \lim_{x\to -3} f(x)$$



d) $f(x) = \dfrac{x^4}{x^3+3x^2}$ en 0 i -3



$$\lim_{x\to 0} \frac{x^4}{x^2(x+3)} = \lim_{x\to 0} \frac{x^2}{x+3} = \frac{0}{3} = 0$$

$$\left.\begin{array}{l} \lim\limits_{x\to -3^-} f(x) = \lim\limits_{x\to -3^-} \dfrac{x^4}{x^3+3x^2} = \dfrac{81}{-27+27} = \dfrac{81}{0^-} = -\infty \\ \lim\limits_{x\to -3^+} f(x) = \lim\limits_{x\to -3^+} \dfrac{x^4}{x^3+3x^2} = \dfrac{81}{0^+} = +\infty \end{array}\right\} \text{No existeix } \lim_{x\to -3} f(x)$$

Pàg 138

8 $\lim\limits_{x\to +\infty} f_1(x) = -\infty$; $\lim\limits_{x\to +\infty} f_2(x) = -3$; $\lim\limits_{x\to +\infty} f_3(x) = +\infty$; $\lim\limits_{x\to +\infty} f_4(x)$ No existeix

Pàg 139

9 a) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} -x^2+3x+5 = -\infty$ b) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} 5x^3+7x = +\infty$

c) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} x-3x^4 = -\infty$ d) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \dfrac{1}{3x} = 0$

e) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} -\dfrac{1}{x^2} = 0$ f) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \dfrac{x^3-1}{-5} = -\infty$

10 $x = 1000 \rightarrow f(x) = x^3 - 200x^2$

$$f(1000) = 1000^3 - 200 \cdot 100^2 = 800\,000\,000$$

(11) $x = 1000 \rightarrow f(x) = \dfrac{1}{x^2 - 10x}$

$$f(1000) = \frac{1}{1000^2 - 10 \cdot 1000} = 0'00000101$$

Pàg 140

(12) a) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{1}{3x} = 0$ 

b) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{3}{x} = 0$ 

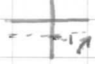
c) $\lim\limits_{x \to +\infty} -\dfrac{1}{x^2} = 0$ 

d) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} 3x - 5 = +\infty$ 

(13) a) $\lim\limits_{x \to +\infty} \dfrac{x^3 - 1}{-5} = -\infty$ 

b) $\lim\limits_{x \to +\infty} \dfrac{x^2 - 3}{x^3} = 0$ 

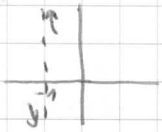
c) $\lim\limits_{x \to +\infty} \dfrac{x^3}{x^2 - 3} = +\infty$ 

d) $\lim\limits_{x \to +\infty} \dfrac{1 - x^3}{1 + x^3} = -1$ 

Pàg 141

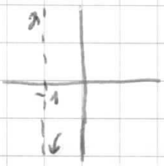
(14) a) $y = \dfrac{x^2 + 3x + 11}{x + 1}$ $\quad x + 1 = 0 \rightarrow x = -1$

$$\left.\begin{aligned} &\lim_{x \to -1^-} \frac{x^3 + 3x + 11}{x + 1} = \frac{7}{0^-} = -\infty \\ &\lim_{x \to -1^+} \frac{x^3 + 3x + 11}{x + 1} = \frac{7}{0^+} = +\infty \end{aligned}\right\} x = -1 \text{ és asímptota vertical}$$



b) $y = \dfrac{x^2 + 3x}{x + 1}$ $\quad x + 1 = 0 \rightarrow x = -1$

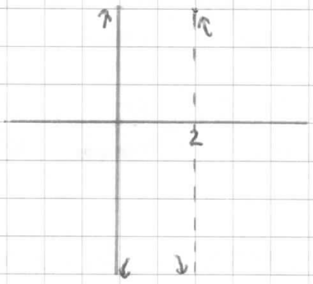
$$\left.\begin{aligned} &\lim_{x \to -1^-} \frac{x^2 + 3x}{x + 1} = \frac{-2}{0^-} = +\infty \\ &\lim_{x \to -1^+} \frac{x^2 + 3x}{x + 1} = \frac{-2}{0^+} = -\infty \end{aligned}\right\} x = -1 \text{ és asímptota vertical}$$



(15) a) $y = \dfrac{x^2 + 2}{x^2 - 2x}$ $\quad x^2 - 2x = 0 < \begin{matrix} x = 0 \\ x - 2 = 0 \rightarrow x = 2 \end{matrix}$

$$\left.\begin{aligned} &\lim_{x \to 0^-} \frac{x^2 + 2}{x^2 - 2x} = \frac{2}{0^+} = +\infty \\ &\lim_{x \to 0^+} \frac{x^2 + 2}{x^2 - 2x} = \frac{2}{0^-} = -\infty \end{aligned}\right\} x = 0 \text{ és asímptota vertical}$$

$$\left.\begin{aligned} &\lim_{x \to 2^-} \frac{x^2 + 2}{x^2 - 2x} = \frac{2}{0^-} = -\infty \\ &\lim_{x \to 2^+} \frac{x^2 + 2}{x^2 - 2x} = \frac{2}{0^+} = +\infty \end{aligned}\right\} x = 2 \text{ és asímptota vertical}$$



b) $y = \dfrac{x^2+2}{x^2-2x+1}$

$x^2-2x+1=0 \rightarrow (x-1)^2=0 \rightarrow x=1$

$\lim\limits_{x\to 1^-} \dfrac{x^2+2}{x^2-2x+1} = \dfrac{3}{0^+} = +\infty$

$\lim\limits_{x\to 1^+} \dfrac{x^2+2}{x^2-2x+1} = \dfrac{3}{0^+} = +\infty$

$\Bigg\}$ $x=1$ és asímptota vertical

Pàg 143

16 a) $y = \dfrac{x}{1+x^2}$

$\lim\limits_{x\to +\infty} \dfrac{x}{1+x^2} = 0$

$y=0$ és una A.H.

b) $y = \dfrac{x^3}{1+x^2}$

$\lim\limits_{x\to +\infty} \dfrac{x^3}{1+x^2} = +\infty$

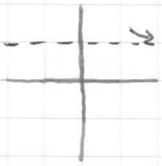
$\lim\limits_{x\to +\infty} \dfrac{\frac{x^3}{1+x^2}}{x} = \lim\limits_{x\to +\infty} \dfrac{x^3}{x+x^3} = 1 \rightarrow \boxed{m=1}$

$\lim\limits_{x\to +\infty} \dfrac{x^3}{1+x^2} - x = \lim\limits_{x\to +\infty} \dfrac{x^3-x-x^3}{1+x^2} = \lim\limits_{x\to +\infty} \dfrac{-x}{1+x^2} = 0 \rightarrow \boxed{n=0}$

$y = mx+n \rightarrow y=x$ és A.O.

17 a) $y = \dfrac{x^2+2}{x^2-2x}$

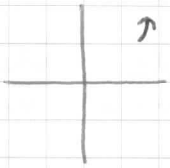
$\lim\limits_{x\to +\infty} \dfrac{x^2+2}{x^2-2x} = 1$



$y=1$ és una A.H.

b) $y = \dfrac{2x^3-3x^2+7}{x}$ grau de P − grau de Q ≥ 2

$\lim\limits_{x\to +\infty} \dfrac{2x^3-3x^2+7}{x} = +\infty$



branca parabòlica cap amunt

Pàg 144

18 $\lim\limits_{x\to -\infty} -2x^3+7x^4-3 = +\infty$

19 a) $\lim\limits_{x\to -\infty} \dfrac{x^2+3}{-x^3} = 0$

b) $\lim\limits_{x\to -\infty} \dfrac{-x^3}{x^2+3} = +\infty$

Pàg 145

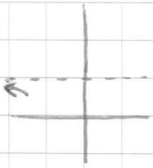
20 a) $\lim\limits_{x\to -\infty} \dfrac{1}{x^2+1} = 0$

$y=0$ A.H.

b) $\lim\limits_{x\to -\infty} \dfrac{x}{1+x^2} = 0$

$y=0$ A.H

c) $\lim_{x \to -\infty} \frac{x^2}{1+x^2} = 1$



$y = 1$ és A.H

d) $\lim_{x \to -\infty} \frac{x^3}{1+x^2} = -\infty$

$\lim_{x \to -\infty} \frac{\frac{x^3}{1+x^2}}{x} = \lim_{x \to -\infty} \frac{x^3}{x+x^3} = 1 \to \boxed{m=1}$

$\lim_{x \to -\infty} \frac{x^3}{1+x^2} - x = \lim_{x \to -\infty} \frac{x^3 - x - x^3}{1+x^2} =$

$= \lim_{x \to -\infty} \frac{-x}{1+x^2} = 0 \to n = 0$

$y = x$ és A.O.

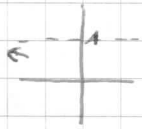
(21) a) $y = \frac{x^4}{x^2+1}$

$\lim_{x \to -\infty} \frac{x^4}{x^2+1} = +\infty$ branca parabòlica

gr x^4 - gr $(x^2+1) = 4 - 2 = 2$

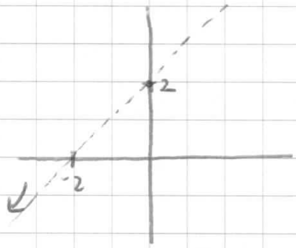
b) $y = \frac{x^2+2}{x^2-2x}$

$\lim_{x \to -\infty} \frac{x^2+2}{x^2-2x} = 1$



$y = 1$ és A.H

c) $y = \frac{x^2+3x}{x+1}$



$\lim_{x \to -\infty} \frac{x^2+3x}{x+1} = -\infty$

$\lim_{x \to -\infty} \frac{\frac{x^2+3x}{x+1}}{x} = \lim_{x \to -\infty} \frac{x^2+3x}{x^2+x} = 1 \to \boxed{m=1}$

$\lim_{x \to -\infty} \frac{x^2+3x}{x+1} - x = \lim_{x \to -\infty} \frac{x^2+3x-x^2-x}{x+1} = \lim_{x \to -\infty} \frac{2x}{x+1} = 2 \to \boxed{n=2}$

$\Big\}\ y = x+2$

d) $y = \frac{2x^3-3x^2}{x}$ gr $(2x^3-3x^2)$ - gr $x = 3 - 1 = 2$

$\lim_{x \to -\infty} \frac{2x^3-3x^2}{x} = +\infty$

Pàg 151

(22) a) gràfica a,

b) gràfica b → Branca infinita en $x=1$ (A.V.)

gràfica c → Branca infinita en $x=0$ (A.V.)

gràfica d → Salt en $x=2$

gràfica e → Punt desplaçat en $x=1$; $f(1)=4$, $\lim_{x \to 1} f(x) = 2$

gràfica f → No està definida en $x=2$

(23) a) $y = x^2+x-6$ contínua

b) $y = \frac{x}{(x-2)^2}$ discontínua en $x=2$

c) $y = \frac{x-1}{2x+1}$ discontínua en $x = -\frac{1}{2}$

d) $y = \frac{1}{x^2+2x+3}$ Contínua

$x = \frac{-2 \pm \sqrt{2^2-4\cdot1\cdot3}}{2\cdot1} = \frac{-2\pm\sqrt{-8}}{2}$ No té solució

e) $y = \frac{2}{5x-x^2}$ discontínua en $x=0$ i $x=5$

$5x-x^2=0 \rightarrow (5-x)x=0 < \begin{array}{l} x=0 \\ 5-x=0 \rightarrow x=5 \end{array}$

f) $y = \frac{1}{x^2+2}$ Contínua

$x^2+2=0 \rightarrow x^2=-2 \rightarrow x=\pm\sqrt{-2}$ No té solució

(24) a) $y = \frac{1}{\sqrt{x}}$ No és contínua ni en $x=0$ ni en $x=-2$

b) $y = \frac{x}{x^2-4}$ És contínua en $x=0$, no és contínua en $x=-2$

c) $y = \sqrt{x^2-4}$ No és contínua en $x=0$, però sí és contínua en $x=-2$

d) $y = \sqrt{7-2x}$ És contínua en $x=0$ i en $x=-2$.

(25) a) $y = 5 - \frac{x}{2}$ És contínua a tot $\mathbb{R}$

b) $y = \sqrt{x-3}$ És contínua en $[3,+\infty)$

$x-3 \geq 0 \rightarrow x \geq 3$

c) $y = \frac{1}{x}$ És contínua en $(-\infty,0)\cup(0,+\infty)$

d) $y = \sqrt{-3x}$ És contínua en $(-\infty,0]$

$-3x \geq 0 \rightarrow 0 \geq 3x \rightarrow 0 \geq x$

e) $y = \sqrt{5-2x}$ És contínua en $(-\infty, \frac{5}{2}]$

$5-2x \geq 0 \rightarrow 5 \geq 2x \rightarrow \frac{5}{2} \geq x$

f) $y = x^2-x$ És contínua a tot $\mathbb{R}$

(26) a) És contínua en $x=1$ b) És discontínua en $x=1$ c) És discontínua en $x=1$

(27) $f(x) = \begin{cases} x^2-1 & \text{si } x<0 \\ x-1 & \text{si } x\geq 0 \end{cases}$

$\lim_{x\to0^-} x^2-1=-1$, $\lim_{x\to0^+} x-1=-1 \Rightarrow \left.\begin{array}{l} \lim_{x\to0} f(x)=-1 \\ f(0)=0-1=-1 \end{array}\right\} \lim_{x\to0} f(x)=f(0)$ És contínua en $x=0$.

(28) a) $f(x) = \begin{cases} \frac{3-x}{2} & \text{si } x<-1 \\ 2x+4 & \text{si } x>-1 \end{cases}$

$$\left.\begin{array}{l} \lim_{x\to -1^-} \frac{3-x}{2} = \frac{4}{2} = 2 \\ \lim_{x\to -1^+} 2x+4 = -2+4 = 2 \end{array}\right\} \lim_{x\to -1} f(x) = 2$$

$f(-1)$ no existeix

No és contínua en $x=-1$ perquè no existeix $f(-1)$.

b) $f(x) = \begin{cases} 2-x^2 & \text{si } x<2 \\ \frac{x}{2}-3 & \text{si } x\geq 2 \end{cases}$

$$\left.\begin{array}{l} \lim_{x\to 2^-} 2-x^2 = -2 \\ \lim_{x\to 2^+} \frac{x}{2}-3 = -2 \end{array}\right\} \lim_{x\to 2} f(x) = -2$$

$f(2) = \frac{2}{2} - 3 = -2$

Sí, és contínua en $x=-2$ ja que $\lim_{x\to 2} f(x) = f(2)$

c) $f(x) = \begin{cases} 3x & \text{si } x\leq 1 \\ x+3 & \text{si } x>1 \end{cases}$

$$\left.\begin{array}{l} \lim_{x\to 1^-} 3x = 3 \\ \lim_{x\to 1^+} x+3 = 4 \end{array}\right\} \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) \Rightarrow \text{No és contínua en } x=1$$

Pàg 152

(29) $\lim_{x\to -2} f_1(x) = \lim_{x\to -2} \frac{1}{(x+2)^2} = +\infty$ $\qquad$ $\lim_{x\to -2} f_2(x) = \lim_{x\to -2} \frac{-1}{x+2} = \nexists$

$\begin{cases} \lim_{x\to -2^-} f(x) = +\infty \\ \lim_{x\to -2^+} f(x) = +\infty \end{cases}$ $\qquad$ $\lim_{x\to -2^-} f(x) = +\infty$, $\quad \lim_{x\to -2^+} f(x) = -\infty$

(30) a) $\lim_{x\to -3^-} f(x) = +\infty$ $\qquad$ d) $\lim_{x\to -\infty} f(x) = 0$ $\qquad$ g) $\lim_{x\to +\infty} f(x) = +\infty$

b) $\lim_{x\to -3^+} f(x) = -\infty$ $\qquad$ e) $\lim_{x\to 2^-} f(x) = 0$ $\qquad$ h) $\lim_{x\to -2} f(x) = 0$

c) $\lim_{x\to 0} f(x) = 2$ $\qquad$ f) $\lim_{x\to 2^+} f(x) = 3$

(31) a) $\lim_{x\to 0} \left(5-\frac{x}{2}\right) = 5$

b) $\lim_{x\to -1} (x^3-x) = (-1)^3-(-1) = -1+1=0$

c) $\lim_{x\to 3} \frac{1-x}{x-2} = \frac{1-3}{3-2} = \frac{-2}{1} = -2$

d) $\lim_{x\to 0'5} 2^x = 2^{0'5} = \sqrt{2}$

e) $\lim_{x\to -2} \sqrt{10+x-x^2} = \sqrt{10-2-(-2)^2} = \sqrt{4} = 2$

f) $\lim_{x\to 4} \log_2 x = \log_2 4 = 2$

g) $\lim_{x\to 0} \cos x = \cos 0 = 1$

h) $\lim_{x\to 2} e^x = e^2$

(32) $f(x) = \begin{cases} x^2+1 & \text{si } x<0 \\ x+1 & \text{si } x\geq 0 \end{cases}$

a) $\lim_{x\to -2} f(x) = \lim_{x\to -2} x^2+1 = (-2)^2+1 = 5$

b) $\lim_{x\to 3} f(x) = \lim_{x\to 3} x+1 = 3+1 = 4$

c) $\lim_{x\to 0} f(x) = 1$

$\lim_{x\to 0^-} x^2+1 = 1$; $\lim_{x\to 0^+} x+1 = 1$

(33) a) $\lim_{x\to 0} \frac{4x}{x^2-2x} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 0} \frac{4x}{x(x-2)} = \lim_{x\to 0} \frac{4}{x-2} = \frac{4}{-2} = -2$

b) $\lim_{x\to 0} \frac{2x^2+3x}{x} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 0} \frac{x(2x+3)}{x} = \lim_{x\to 0} 2x+3 = 3$

c) $\lim_{h\to 0} \frac{3h^3-2h^2}{h} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{h\to 0} \frac{h(3h^2-2h)}{h} = \lim_{h\to 0} 3h^2-2h = 0$

d) $\lim_{h\to 0} \frac{h^2-7h}{4h} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{h\to 0} \frac{h(h-7)}{4h} = \lim_{h\to 0} \frac{h-7}{4} = \frac{-7}{4}$

(34) a) $\lim_{x\to 1} \frac{x^2-1}{x-1} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1} x+1 = 2$

b) $\lim_{x\to -1} \frac{x^3+1}{x^2+x} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to -1} \frac{(x+1)(x^2-x+1)}{x(x+1)} = \lim_{x\to -1} \frac{x^2-x+1}{x} = \frac{3}{-1} = -3$

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

c) $\lim\limits_{x \to -2} \dfrac{x+2}{x^2-4} = \lim\limits_{x \to -2} \dfrac{x+2}{(x+2)(x-2)} = \lim\limits_{x \to -2} \dfrac{1}{x-2} = \dfrac{1}{-4} = \dfrac{-1}{4}$

d) $\lim\limits_{x \to 2} \dfrac{x^2-x-2}{x-2} = \lim\limits_{x \to 2} \dfrac{(x-2)(x+1)}{x-2} = \lim\limits_{x \to 2} x+1 = 3$

$$\begin{array}{c|ccc} & 1 & -1 & -2 \\ 2 & & 2 & 2 \\ \hline & 1 & 1 & \boxed{0} \end{array}$$

e) $\lim\limits_{x \to -3} \dfrac{x+3}{x^2+4x+3} = \lim\limits_{x \to -3} \dfrac{x+3}{(x+3)(x+1)} = \lim\limits_{x \to -3} \dfrac{1}{x+1} = \dfrac{1}{-2} = \dfrac{-1}{2}$

$$\begin{array}{c|ccc} & 1 & 4 & 3 \\ -3 & & -3 & -3 \\ \hline & 1 & 1 & \boxed{0} \end{array}$$

f) $\lim\limits_{x \to 1} \dfrac{x^4-1}{x^2-1} = \lim\limits_{x \to 1} \dfrac{(x^2-1)(x^2+1)}{x^2-1} = \lim\limits_{x \to 1} x^2+1 = 2$

35) $f(x) = \dfrac{x^2}{x^2+x}$

$\lim\limits_{x \to 3} \dfrac{x^2}{x^2+x} = \dfrac{9}{9+3} = \dfrac{9}{12} = \dfrac{3}{4}$

$\lim\limits_{x \to 0} \dfrac{x^2}{x^2+x} = \left(\dfrac{0}{0} \text{ IND} \right) = \lim\limits_{x \to 0} \dfrac{x^2}{x(x+1)} = \lim\limits_{x \to 0} \dfrac{x}{x+1} = \dfrac{0}{1} = 0$

$\lim\limits_{x \to -1} \dfrac{x^2}{x^2+x}$ No existeix

↳ $\lim\limits_{x \to -1^-} \dfrac{x^2}{x^2+x} = \dfrac{1}{0^+} = +\infty$

$\lim\limits_{x \to -1^+} \dfrac{x^2}{x^2+x} = \dfrac{1}{0^-} = -\infty$

36)
a) $\lim\limits_{x \to +\infty} (7+x-x^3) = -\infty$

b) $\lim\limits_{x \to +\infty} \dfrac{x^2-10x-32}{5} = +\infty$

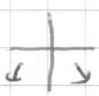
c) $\lim\limits_{x \to +\infty} \left(-\dfrac{x^4}{3} + \dfrac{x}{2} - 17 \right) = -\infty$

d) $\lim\limits_{x \to +\infty} (7-x)^2 = +\infty$

37)
a) $\lim\limits_{x \to -\infty} (7+x-x^3) = +\infty$

b) $\lim\limits_{x \to -\infty} \dfrac{x^2-10x-32}{5} = +\infty$

c) $\lim\limits_{x \to -\infty} \left(\dfrac{-x^4}{3} + \dfrac{x}{2} - 17 \right) = -\infty$



d) $\lim\limits_{x \to -\infty} (7-x)^2 = +\infty$

(38) a) $f(x)=\dfrac{1}{x^2-10}$ b) $f(x)=\dfrac{100}{3x^2}$ c) $f(x)=\dfrac{-7}{\sqrt{x}}$ d) $f(x)=\dfrac{2}{10x^2-x^3}$

$\lim\limits_{x\to+\infty} f(x)=0$ $\lim\limits_{x\to+\infty} f(x)=0$ $\lim\limits_{x\to\infty} f(x)=0$ $\lim\limits_{x\to+\infty} f(x)=0$

(39) a) $f(x)=x^3-10x$ b) $f(x)=\sqrt{x^2-4}$ c) $f(x)=\dfrac{3-x}{2}$

$\lim\limits_{x\to-\infty} x^3-10x=-\infty$ $\lim\limits_{x\to-\infty}\sqrt{x^2-4}=+\infty$ $\lim\limits_{x\to-\infty}\dfrac{3-x}{2}=+\infty$

$\lim\limits_{x\to+\infty} x^3-10x=+\infty$ $\lim\limits_{x\to+\infty}\sqrt{x^2-4}=+\infty$ $\lim\limits_{x\to+\infty}\dfrac{3-x}{2}=-\infty$

d) $f(x)=\dfrac{x^2-2x}{-3}$

$\lim\limits_{x\to-\infty}\dfrac{x^2-2x}{-3}=+\infty$

$\lim\limits_{x\to+\infty}\dfrac{x^2-2x}{-3}=-\infty$

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(40) a) $\lim\limits_{x\to+\infty}\dfrac{3}{(x-1)^2}=0$

b) $\lim\limits_{x\to+\infty}\dfrac{-2x^2}{3-x}=+\infty$

c) $\lim\limits_{x\to+\infty}\dfrac{-1}{x^2-1}=0$

d) $\lim\limits_{x\to+\infty}\dfrac{1}{(2-x)^3}=0$

e) $\lim\limits_{x\to+\infty}\dfrac{2x-1}{x+2}=2$

f) $\lim\limits_{x\to+\infty}\dfrac{x^2+5}{1-x}=-\infty$

g) $\lim\limits_{x\to+\infty}\dfrac{2-3x}{x+3}=-3$

h) $\lim\limits_{x\to+\infty}\dfrac{3-2x}{5-2x}=1$

(41) a) $\lim\limits_{x\to-\infty}\dfrac{3}{(x-1)^2}=0$

b) $\lim\limits_{x\to-\infty}\dfrac{-2x^2}{3-x}=-\infty$

c) $\lim\limits_{x\to-\infty}\dfrac{-1}{x^2-1}=0$

d) $\lim\limits_{x\to-\infty}\dfrac{1}{(2-x)^3}=0$

e) $\lim\limits_{x\to-\infty}\dfrac{2x-1}{x+2}=2$

f) $\lim\limits_{x\to-\infty}\dfrac{x^2+5}{1-x}=+\infty$

g) $\lim\limits_{x\to-\infty}\dfrac{2-3x}{x+3}=-3$

h) $\lim\limits_{x\to-\infty}\dfrac{3-2x}{5-2x}=1$

(42) a) $\lim\limits_{x\to+\infty} \dfrac{3x^2}{(x-1)^2} = 3$ b) $\lim\limits_{x\to-\infty} 1-(x-2)^2 = -\infty$

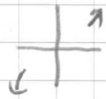
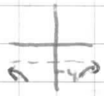
c) $\lim\limits_{x\to+\infty} \dfrac{1-x}{(2x+1)^2} = 0$ d) $\lim\limits_{x\to-\infty} \dfrac{x^3+1}{5x} = +\infty$

(43) a) $f(x) = \dfrac{-1}{x^2}$ b) $f(x) = 10x - x^3$ c) $f(x) = \dfrac{x^2}{x-1}$ d) $f(x) = \dfrac{1-12x^2}{3x^2}$

$\lim\limits_{x\to-\infty} \dfrac{-1}{x^2} = 0$ $\lim\limits_{x\to-\infty} 10x-x^3 = +\infty$ $\lim\limits_{x\to-\infty} \dfrac{x^2}{x-1} = -\infty$ $\lim\limits_{x\to-\infty} \dfrac{1-12x^2}{3x^2} = -4$

$\lim\limits_{x\to+\infty} \dfrac{-1}{x^2} = 0$ $\lim\limits_{x\to+\infty} 10x-x^3 = -\infty$ $\lim\limits_{x\to+\infty} \dfrac{x^2}{x-1} = +\infty$ $\lim\limits_{x\to+\infty} \dfrac{1-12x^2}{3x^2} = -4$

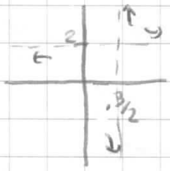


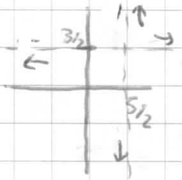



(44) a) $y = \dfrac{4x+1}{2x-3}$ b) $y = \dfrac{3x}{2x-5}$

$2x-3=0 \to x = \dfrac{3}{2} \Rightarrow$ A.V: $x = \dfrac{3}{2}$ $2x-5=0 \to x=\dfrac{5}{2} \Rightarrow$ A.V: $x=\dfrac{5}{2}$

$\lim\limits_{x\to\pm\infty} \dfrac{4x+1}{2x-3} = \dfrac{4}{2} = 2 \Rightarrow$ A.H: $y=2$ $\lim\limits_{x\to\pm\infty} \dfrac{3x}{2x-5} = \dfrac{3}{2} \Rightarrow$ A.H: $y = \dfrac{3}{2}$

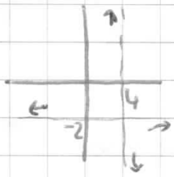


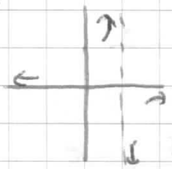


c) $y = \dfrac{2x+3}{4-x}$ d) $y = \dfrac{2}{1-x}$

$4-x=0 \to x=4 \Rightarrow$ A.V: $x=4$ $1-x=0 \to x=1 \Rightarrow$ A.V: $x=1$

$\lim\limits_{x\to\pm\infty} \dfrac{2x+3}{4-x} = -2 \Rightarrow$ AH: $y=-2$ $\lim\limits_{x\to\pm\infty} \dfrac{2}{1-x} = 0 \Rightarrow$ A.H: $y=0$





(45) a) $y = \dfrac{x^2}{x^2+4}$ b) $y = \dfrac{3}{x^2+1}$

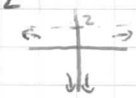
$x^2+4=0 \to x = \pm\sqrt{-4}$ No té solució $x^2+1=0 \to x=\pm\sqrt{-1}$ No té solució

$\lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2+4} = 1 \Rightarrow$ AH: $y=1$ $\lim\limits_{x\to\pm\infty} \dfrac{3}{x^2+1} = 0 \Rightarrow$ A.H: $y=0$


c) $y = \frac{2x^2-1}{x^2}$

$x^2 = 0 \rightarrow x = 0 \Rightarrow$ A.V: $x=0$

$\lim_{x\to\pm\infty} \frac{2x^2-1}{x^2} = 2 \Rightarrow$ A.H: $y=2$



d) $y = \frac{x^4}{x-1}$

$x-1=0 \rightarrow x=1$

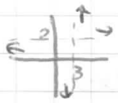
$\lim_{x\to+\infty} \frac{x^4}{x-1} = +\infty$

$\lim_{x\to-\infty} \frac{x^4}{x-1} = -\infty$

46 a) $f(x) = \frac{2x}{x-3}$

$x-3=0 \rightarrow x=3 \Rightarrow$ A.V: $x=3$

$\lim_{x\to\pm\infty} \frac{2x}{x-3} = 2 \Rightarrow$ A.H: $y=2$



b) $f(x) = \frac{x-1}{x+3}$

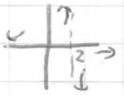
$x+3=0 \rightarrow x=-3 \Rightarrow$ A.V.: $x=-3$

$\lim_{x\to\pm\infty} \frac{x-1}{x+3} = 1$

c) $f(x) = \frac{1}{2-x}$

$2-x=0 \rightarrow x=2 \Rightarrow$ AV: $x=2$

$\lim_{x\to\pm\infty} \frac{1}{2-x} = 0 \Rightarrow$ A.H: $y=0$



d) $f(x) = \frac{1}{x^2+9}$

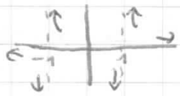
$x^2+9=0 \rightarrow x = \pm\sqrt{-9}$ No té solució

$\lim_{x\to\pm\infty} \frac{1}{x^2+9} = 0 \Rightarrow$ A.H: $y=0$

e) $f(x) = \frac{3x}{x^2-1}$

$x^2-1=0 \rightarrow x=\pm1 \Rightarrow$ A.V: $x=1$, $x=-1$

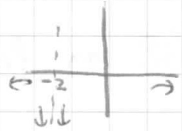
$\lim_{x\to\pm\infty} \frac{3x}{x^2-1} = 0 \Rightarrow$ A.H: $y=0$



f) $f(x) = \frac{-1}{(x+2)^2}$

$x+2=0 \rightarrow x=-2 \Rightarrow$ A.V: $x=-2$

$\lim_{x\to\pm\infty} \frac{-1}{(x+2)^2} = 0 \Rightarrow$ A.H: $y=0$

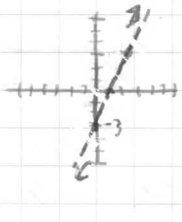


47 a) $f(x) = \frac{3x^2}{x+1}$

Grau$(3x^2)$ − gr$(x+1) = 2-1 = 1$.

$$\begin{array}{l|l} 3x^2 & x+1 \\ \hline -3x^2-3x & 3x-3 \\ \quad / \ -3x & \\ \quad\ \ 3x+3 & \\ \quad\ \ / \ \ 3 & \end{array}$$

$\frac{3x^2}{x+1} = 3x-3 + \frac{3}{x+1}$ $\Rightarrow$ A.O: $y = 3x-3$



b) $f(x) = \frac{3+x-x^2}{x}$

gr$(3+x-x^2)$ − gr$(x) = 2-1 = 1$

$\frac{3+x-x^2}{x} = -x+1 + \frac{3}{x}$

$$\begin{array}{l|l} -x^2+x+3 & x \\ \hline x^2 & -x+1 \\ / \quad x+3 & \\ \quad\ -x & \\ \quad\ / \ \ 3 & \end{array}$$

A.O: $y = -x+1$

c) $f(x) = \dfrac{4x^2-3}{2x}$ $\quad gr(4x^2-3) + gr(2x) = 2-1=1$

$$\begin{array}{l|l} 4x^2-3 & 2x \\ \hline -4x^2 & 2x \\ / \quad -3 & \end{array} \Rightarrow \frac{4x^2-3}{2x} = 2x + \frac{-3}{2x} \Rightarrow A.O: y = 2x$$

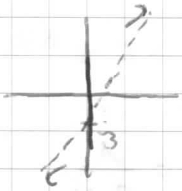
d) $f(x) = \dfrac{x^2+x-2}{x-3}$ $\quad gr(x^2+x-2) - gr(x-3) = 2-1=1$

$$\begin{array}{l|l} x^2+x-2 & x-3 \\ \hline -x^2+3x & x+4 \\ / \quad 4x-2 & \\ \quad -4x+12 & \\ \quad / \quad 10 & \end{array} \Rightarrow \frac{x^2+x-2}{x-3} = x+4+\frac{10}{x-3}$$

$A.O: y = x+4$

e) $f(x) = \dfrac{2x^3-3}{x^2-2}$ $\quad gr(2x^3-3) - gr(x^2-2) = 3-2=1$

$$\begin{array}{l|l} 2x^3 \quad -3 & x^2-2 \\ \hline -2x^3+4x & 2x \\ / \quad 4x-3 & \end{array} \Rightarrow \frac{2x^3-3}{x^2-2} = 2x + \frac{4x-3}{x^2-2}$$



$AO: y = 2x$

f) $f(x) = \dfrac{-2x^2+3}{2x-2}$ $\quad gr(-2x^2+3) - gr(2x-2) = 2-1=1$

$$\begin{array}{l|l} -2x^2 \quad +3 & 2x-2 \\ \hline 2x^2-2x & -x-1 \\ / \quad -2x+3 & \\ \quad 2x-2 & \\ \quad / \quad 1 & \end{array} \Rightarrow \frac{-2x^2+3}{2x-2} = -x-1+\frac{1}{2x-2}$$

$A.O: y = -x-1$

48 a) $f(x) = \dfrac{3x}{2x+4}$ $\quad 2x+4=0 \rightarrow 2x=-4 \rightarrow x = \frac{-4}{2} \rightarrow x=-2$

$$\lim_{x\to -2^-} \frac{3x}{2x+4} = \frac{-6}{0^-} = +\infty \qquad \lim_{x\to -2^+} \frac{3x}{2x+4} = \frac{-6}{0^+} = -\infty \quad \Big\} \lim_{x\to -2} f(x) \nexists$$

b) $y = \dfrac{x-1}{x^2-2x}$ $\quad x^2-2x=0 \rightarrow x(x-2)=0 \rightarrow x=0; \quad x-2=0 \rightarrow x=2$

$$\lim_{x\to 0^-} \frac{x-1}{x^2-2x} = \frac{-1}{0^+} = -\infty \qquad \lim_{x\to 0^+} \frac{x-1}{x^2-2x} = \frac{-1}{0^-} = +\infty \quad \Big\} \lim_{x\to 0} f(x) \nexists$$

$$\lim_{x\to 2^-} \frac{x-1}{x^2-2x} = \frac{1}{0^+} = +\infty \qquad \lim_{x\to 2^+} \frac{x-1}{x^2-2x} = \frac{1}{0^+} = +\infty \quad \Big\} \lim_{x\to 2} f(x) \nexists$$

c) $f(x) = \dfrac{x^2-2x}{x^2-4}$ $\quad x^2-4=0 \rightarrow x^2=4 \rightarrow x = \pm\sqrt{4} \rightarrow x = \pm 2$

$$\lim_{x\to -2^-} \frac{x^2-2x}{x^2-4} = \frac{8}{0^+} = +\infty \qquad \lim_{x\to -2^+} \frac{x^2-2x}{x^2-4} = \frac{8}{0^-} = -\infty$$

$$\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 2^-} \frac{x(x-2)}{(x-2)(x+2)} = \lim_{x\to 2^-} \frac{x}{x+2} = \frac{2}{4} = \frac{1}{2} \quad \Big\}$$

$$\lim_{x\to 2^+} \frac{x^2-2x}{x^2-4} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 2^+} \frac{x(x-2)}{(x-2)(x+2)} = \lim_{x\to 2^+} \frac{x}{x+2} = \frac{2}{4} = \frac{1}{2} \Bigg\} \Rightarrow$$

$$\Rightarrow \lim_{x\to 2} f(x) = \frac{1}{2}$$

d) $f(t) = \dfrac{t^3-2t^2}{t^2}$ $\quad t^2=0 \to t=0$

$$\lim_{x\to 0} \frac{t^3-2t^2}{t^2} = \left(\frac{0}{0}\text{ IND}\right) \lim_{x\to 0} \frac{t^2(t-2)}{t^2} = \lim_{x\to 0} t-2 = -2$$

$$\lim_{x\to 0^-} t-2 = -2 \ , \ \lim_{x\to 0^+} t-2 = -2$$

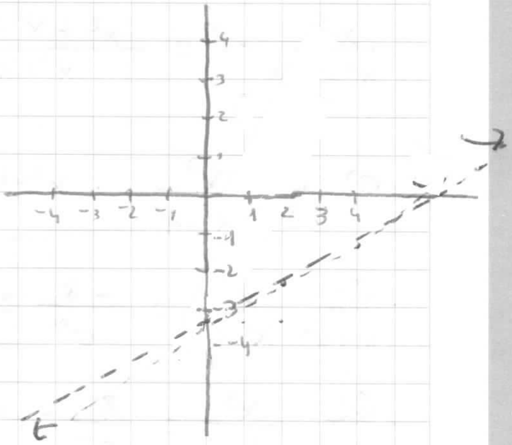
49 a) $y = \dfrac{(3-x)^2}{2x+1}$ $\quad 2x+1=0 \to 2x=-1 \to x = \dfrac{-1}{2}$

$$\lim_{x\to -\frac{1}{2}^-} \frac{(3-x)^2}{2x+1} = \frac{\left(\frac{7}{2}\right)^2}{0^+} = +\infty \ ; \ \lim_{x\to -\frac{1}{2}^+} \frac{(3-x)^2}{2x+1} = \frac{\left(\frac{7}{2}\right)^2}{0^-} = -\infty \Rightarrow \text{A.V}: x = -\frac{1}{2}$$

$gr[(3-x)^2] - gr(2x) = 2-1 = 1$

$$\begin{array}{l|l} x^2-6x+9 & 2x+1 \\ \hline -x^2 - \frac{1}{2}x & \frac{1}{2}x - \frac{13}{4} \\ \hline / \ -\frac{13}{2}x+9 & \\ \quad \frac{13}{2}x + \frac{13}{4} & \\ \hline \quad / \quad \frac{49}{4} & \end{array}$$

$\Rightarrow$ A.O: $y = \frac{1}{2}x - \frac{13}{4}$

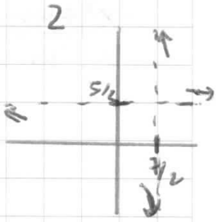


b) $y = \dfrac{5x-2}{2x-7}$ $\quad 2x-7=0 \to x = \dfrac{7}{2}$

$$\frac{5x-2}{2x-7} - \frac{5}{2} = \frac{10x-4-10x+35}{5x-2} = \frac{31}{5x-2}$$

$$\lim_{x\to \frac{7}{2}^-} \frac{5x-2}{2x-7} = \frac{\frac{31}{2}}{0^-} = -\infty \quad \lim_{x\to \frac{7}{2}^+} \frac{5x-2}{2x-7} = \frac{\frac{31}{2}}{0^+} = +\infty \Rightarrow \text{A.V}: x = \frac{7}{2}$$

$$\lim_{x\to +\infty} \frac{5x-2}{2x-7} = \frac{5}{2} \ , \ \lim_{x\to -\infty} \frac{5x-2}{2x-7} = \frac{5}{2} \Rightarrow \text{A.H}: y = \frac{5}{2}$$



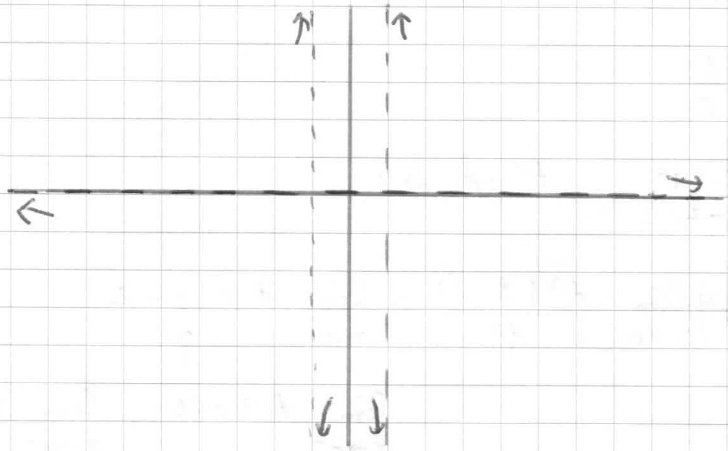
c) $y = \dfrac{x+2}{x^2-1}$ $\quad x^2-1=0 \to x = \pm\sqrt{1} \to x = \pm 1$

$$\lim_{x\to -1^-} \frac{x+2}{x^2-1} = \frac{1}{0^+} = +\infty \ , \ \lim_{x\to -1^+} \frac{1}{0^-} = -\infty \Rightarrow \text{AV}: x = -1$$

$$\lim_{x\to 1^-} \frac{x+2}{x^2-1} = \frac{3}{0^-} = -\infty \ , \ \lim_{x\to 1^+} \frac{3}{0^+} = +\infty \Rightarrow \text{A.V}: x = 1$$

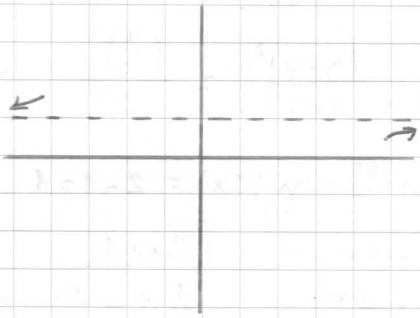
$\lim\limits_{x \to +\infty} \dfrac{x+2}{x^2-1} = 0$, $\lim\limits_{x \to -\infty} \dfrac{x+2}{x^2-1} = 0$ $\Rightarrow$ A.H: $y=0$

$\dfrac{x+2}{x^2-1} - 0 = \dfrac{x+2}{x^2-1}$



d) $y = \dfrac{x^2}{x^2+x+1}$ $x^2+x+1=0$ $x = \dfrac{-1 \pm \sqrt{1^2-4\cdot 1\cdot 1}}{2\cdot 1} = \dfrac{-1 \pm \sqrt{-3}}{2}$ No té solució real

- No té A.V.
- $\lim\limits_{x \to +\infty} \dfrac{x^2}{x^2+x+1} = 1$, $\lim\limits_{x \to -\infty} \dfrac{x^2}{x^2+x+1} = 1$



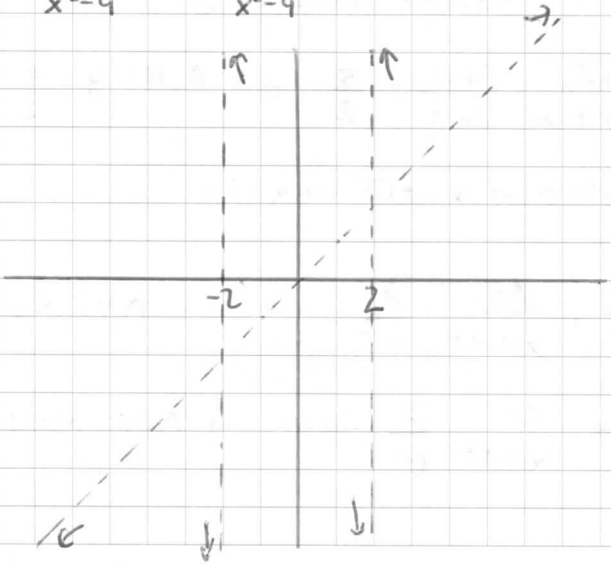
$\dfrac{x^2}{x^2+x+1} - 1 = \dfrac{x^2-x^2-x-1}{x^2+x+1} = \dfrac{-x-1}{x^2+x+1}$

e) $y = \dfrac{x^3}{x^2-4}$ $x^2-4=0 \to x^2=4 \to x = \pm\sqrt{4} \to x = \pm 2$

- $\lim\limits_{x \to -2^-} \dfrac{x^3}{x^2-4} = \dfrac{-8}{0^+} = -\infty$ $\lim\limits_{x \to -2^+} \dfrac{x^3}{x^2-4} = \dfrac{-8}{0^-} = +\infty \Rightarrow$ A.V: $x=-2$
- $\lim\limits_{x \to 2^-} \dfrac{x^3}{x^2-4} = \dfrac{8}{0^-} = -\infty$ $\lim\limits_{x \to 2^+} \dfrac{x^3}{x^2-4} = \dfrac{8}{0^+} = +\infty \Rightarrow$ A.V: $x=2$

gr (x^3) - gr $(x^2-4) = 3-2 = 1$

$$\begin{array}{l|l} x^3 & x^2-4 \\ \cline{2-2} -x^3+4x & x \\ \hline / \quad 4x & \end{array} \Rightarrow \frac{x^3}{x^2-4} = x + \frac{4x}{x^2-4} \Rightarrow \text{A.O: } y = x$$

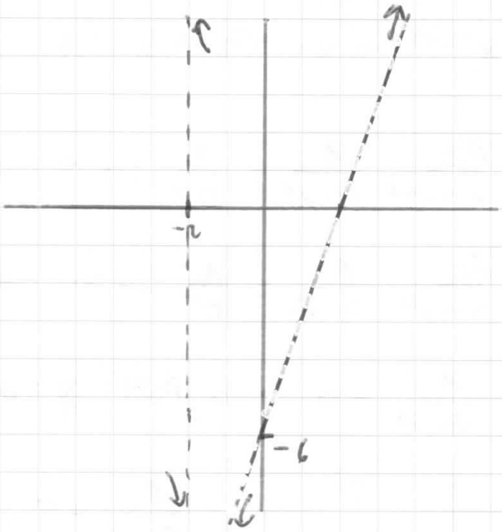


f) $y = \dfrac{3x^2}{x+2}$ $\quad x+2=0 \rightarrow x=-2$

. $\lim\limits_{x\to -2^-} \dfrac{3x^2}{x+2} = \dfrac{12}{0^-} = -\infty$, $\lim\limits_{x\to -2^+} \dfrac{3x^2}{x+2} = \dfrac{12}{0^+} = +\infty \Rightarrow$ A.V: $x=-2$

$gr(3x^2) - gr(x+2) = 2-1 = 1$

. $\begin{array}{l|l} 3x^2 & x+2 \\ \hline -3x^2-6x & 3x-6 \\ \quad / \; -6x & \\ \quad\; 6x+12 & \\ \quad\quad / \; 12 & \end{array} \Rightarrow \dfrac{3x^2}{x+2} = 3x-6+\dfrac{12}{x+2} \Rightarrow$ A.O: $y=3x-6$

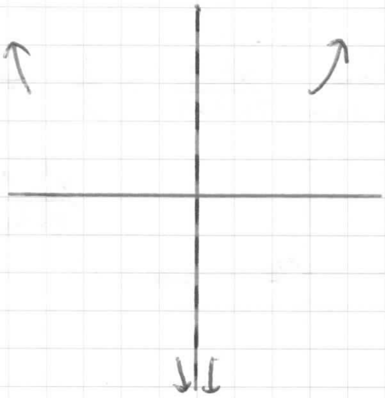


(50) a) $y = \dfrac{x^4-1}{x^2}$ $\quad x^2=0 \rightarrow x=0$

. $\lim\limits_{x\to 0^-} \dfrac{x^4-1}{x^2} = \dfrac{-1}{0^+} = -\infty$ $\quad \lim\limits_{x\to 0^+} \dfrac{x^4-1}{x^2} = \dfrac{-1}{0^+} = +\infty \Rightarrow$ A.V: $x=0$

. $gr(x^4-1) - gr(x^2) = 4-2 = 2$ $\Big\} \Rightarrow$ Hi ha una branca parabòlica, cap amunt

$\lim\limits_{x\to +\infty} \dfrac{x^4-1}{x^2} = +\infty$

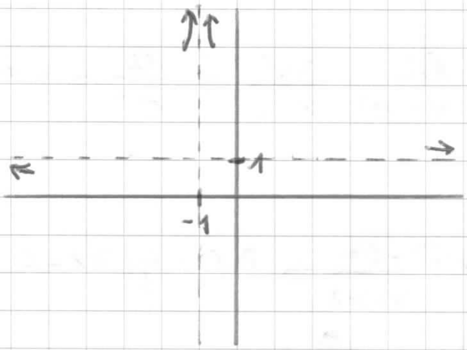


b) $y = \dfrac{(x+3)^2}{(x+1)^2}$ $\quad (x+1)^2=0 \rightarrow x+1=0 \rightarrow x=-1$

. $\lim\limits_{x\to -1} \dfrac{(x+3)^2}{(x+1)^2} = \dfrac{4}{0^+} = +\infty$, $\lim\limits_{x\to -1} \dfrac{(x+3)^2}{(x+1)^2} = \dfrac{4}{0^+} = +\infty \Rightarrow$ A.V: $x=-1$

. $\lim\limits_{x\to +\infty} \dfrac{(x+3)^2}{(x+1)^2} = \lim\limits_{x\to +\infty} \dfrac{x^2+6x+9}{x^2+2x+1} = \dfrac{1}{1} = 1$ $\Big\{$

$$\lim_{x\to-\infty} \frac{(x+3)^2}{(x+1)^2} = \lim_{x\to-\infty} \frac{x^2+6x+9}{x^2+x+1} = \frac{1}{1} = 1 \quad \Rightarrow A.H: y=1$$



$$\frac{x^2+6x+9}{x^2+x+1} - 1 =$$

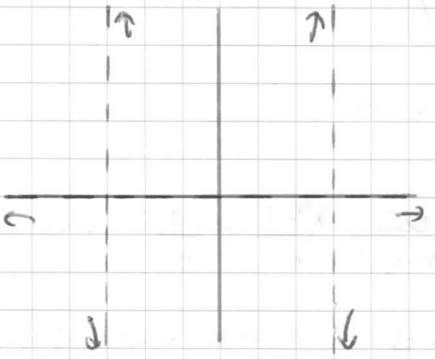
$$= \frac{x^2+6x+9-x^2-x-1}{x^2+x+1} = \frac{5x+8}{x^2+x+1}$$

c) $y = \dfrac{1}{9-x^2}$ $\quad 9-x^2=0 \to x^2=9 \to x=\pm\sqrt{9} \to x=\pm 3$

$$\lim_{x\to-3^-} \frac{1}{9-x^2} = \frac{1}{0^-} = -\infty \qquad \lim_{x\to-3^+} \frac{1}{9-x^2} = \frac{1}{0^+} = +\infty \Rightarrow A.V: x=-3$$

$$\lim_{x\to3^-} \frac{1}{9-x^2} = \frac{1}{0^+} = +\infty \qquad \lim_{x\to3^+} \frac{1}{9-x^2} = \frac{1}{0^-} = -\infty \Rightarrow A.V: x=3$$

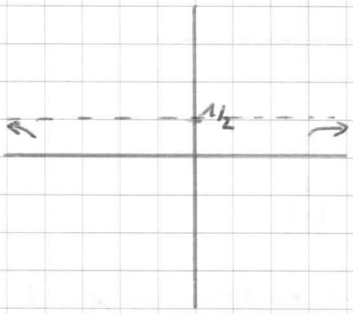
$$\lim_{x\to+\infty} \frac{1}{9-x^2} = 0 \ , \ \lim_{x\to-\infty} \frac{1}{9-x^2} = 0 \Rightarrow A.H: y=0$$



$$\frac{1}{9-x^2} - 0 = \frac{1}{9-x^2}$$

d) $y = \dfrac{x^2-1}{2x^2+1}$ $\quad 2x^2+1=0 \to 2x^2=-1 \to x^2=\dfrac{-1}{2} \to x=\pm\sqrt{\dfrac{-1}{2}}$ No té solució real

$$\lim_{x\to-\infty} \frac{x^2-1}{2x^2+1} = \frac{1}{2} \ , \ \lim_{x\to+\infty} \frac{x^2-1}{2x^2+1} = \frac{1}{2} \Rightarrow A.H: y=\frac{1}{2}$$



$$\frac{x^2-1}{2x^2+1} - \frac{1}{2} = \frac{2x^2-2-2x^2-1}{4x^2+2} =$$

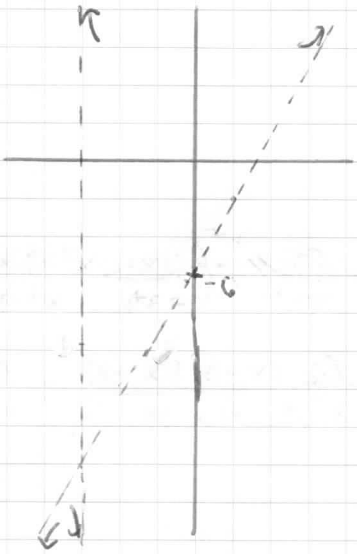
$$= \frac{-3}{4x^2+2}$$

e) $y = \dfrac{2x^2}{x+3}$ $\quad x+3=0 \to x=-3$

$$\lim_{x\to-3^-} \frac{2x^2}{x+3} = \frac{18}{0^-} = -\infty \qquad \lim_{x\to-3^+} \frac{2x^2}{x+3} = \frac{18}{0^+} = +\infty \Rightarrow A.V: x=-3$$

$gr(2x^2) - gr(x+3) = 2-1 = 1$

$$\begin{array}{l|l} 2x^2 & x+3 \\ \hline -2x^2-6x & 2x-6 \\ \quad / \; -6x & \\ \quad\; 6x+18 & \\ \quad / \;\; 18 & \end{array} \Rightarrow \frac{2x^2}{x+3} = 2x-6+\frac{18}{x+3} \Rightarrow A.O.: y = 2x-6$$

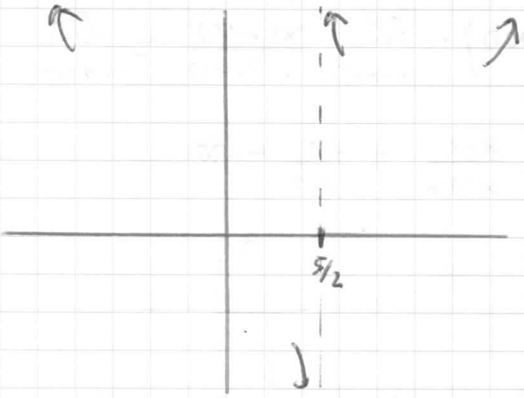


f) $y = \frac{x^3}{2x-5}$ $\qquad 2x-5=0 \rightarrow 2x=5 \rightarrow x=\frac{5}{2}$

$\lim\limits_{x\to\frac{5}{2}^-} \frac{x^3}{2x-5} = \frac{\left(\frac{5}{2}\right)^3}{0^-} = -\infty \qquad \lim\limits_{x\to\frac{5}{2}^+} \frac{x^3}{2x-5} = \frac{\left(\frac{5}{2}\right)^3}{0^+} = +\infty \rightarrow A.V: x=\frac{5}{2}$

$gr(x^3) - gr(2x-5) = 3-1 = 2$

$\lim\limits_{x\to+\infty} \frac{x^3}{2x-5} = +\infty \Rightarrow$ branca parabòlica cap amunt.



Pàg 154

51) $f(x) = \frac{x^2-4}{x^2-2x}$ $\qquad x^2-2x=0 \rightarrow x(x-2)=0 \begin{cases} x=0 \\ x-2=0 \rightarrow x=2 \end{cases}$

$\lim\limits_{x\to0^-} \frac{x^2-4}{x^2-2x} = \frac{-4}{0^+} = -\infty \qquad \lim\limits_{x\to0^+} \frac{x^2-4}{x^2-2x} = \frac{-4}{0^-} = +\infty \Rightarrow A.V: x=0$

$\lim\limits_{x\to2^-} \frac{x^2-4}{x^2-2x} = \left(\frac{0}{0} \text{ IND}\right) = \lim\limits_{x\to2^-} \frac{(x-2)(x+2)}{x(x-2)} = \lim\limits_{x\to2^-} \frac{x+2}{x} = \frac{4}{2} = 2$

$\lim\limits_{x\to2^+} \frac{x^2-4}{x^2-2x} = \left(\frac{0}{0} \text{ IND}\right) = \lim\limits_{x\to2^+} \frac{(x-2)(x+2)}{x(x-2)} = \lim\limits_{x\to2^+} \frac{x+2}{x} = \frac{4}{2} = 2$

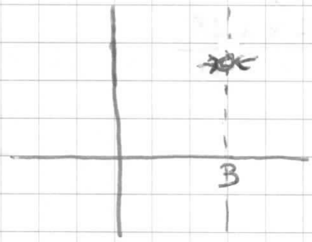
$\lim\limits_{x\to2} f(x) = 2$

$gr(x^2-4) - gr(x^2-2x) = 2-2=0$

$\lim\limits_{x\to+\infty} \dfrac{x^2-4}{x^2-2x} = \dfrac{1}{1} = 1 \qquad \lim\limits_{x\to-\infty} \dfrac{x^2-4}{x^2-2x} = \dfrac{1}{1} = 1 \Rightarrow$ A.H: $y=1$

Asímptota vertical: $x=0$

Asímptota horitzontal: $y=1$



52 a) $\lim\limits_{x\to 3} \dfrac{x^2-x-6}{x^2-3x} = \dfrac{5}{2}$

$\lim\limits_{x\to 3^-} \dfrac{x^2-x-6}{x^2-3x} = \lim\limits_{x\to 3^-} \dfrac{(x-3)(x+2)}{x(x-3)} = \lim\limits_{x\to 3^-} \dfrac{x+2}{x} = \dfrac{5}{3}$

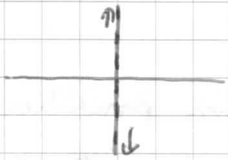
$\lim\limits_{x\to 3^+} \dfrac{x^2-x-6}{x^2-3x} = \lim\limits_{x\to 3^+} \dfrac{(x-3)(x+2)}{x(x-3)} = \lim\limits_{x\to 3^+} \dfrac{x+2}{x} = \dfrac{5}{3}$

b) $\lim\limits_{x\to 1} \dfrac{x^2-3x+2}{x^2-2x+1} = \nexists$

$\lim\limits_{x\to 1^-} \dfrac{x^2-3x+2}{x^2-2x+1} = \lim\limits_{x\to 1^-} \dfrac{(x-1)(x-2)}{(x-1)(x-1)} = \lim\limits_{x\to 1^-} \dfrac{x-2}{x-1} = \dfrac{-1}{0^-} = +\infty$

$\lim\limits_{x\to 1^+} \dfrac{x^2-3x+2}{x^2-2x+1} = \lim\limits_{x\to 1^+} \dfrac{x-2}{x-1} = \dfrac{-1}{0^+} = -\infty$

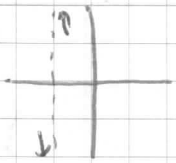
53 a) $\lim\limits_{x\to 0} \dfrac{x^2-2x}{x^3+x^2} \nexists$



$\lim\limits_{x\to 0^-} \dfrac{x^2-2x}{x^3+x^2} = \left(\dfrac{0}{0} \text{ IND}\right) = \lim\limits_{x\to 0^-} \dfrac{x(x-2)}{x^2(x+1)} = \lim\limits_{x\to 0^-} \dfrac{x-2}{x^2+x} = \dfrac{-2}{0^-} = +\infty$

$\lim\limits_{x\to 0^+} \dfrac{x^2-2x}{x^3+x^2} = \lim\limits_{x\to 0^+} \dfrac{x-2}{x^2+x} = \dfrac{-2}{0^+} = -\infty$

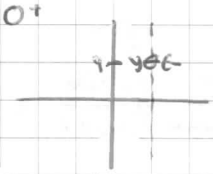
b) $\lim\limits_{x\to -1} \dfrac{x^3+x^2}{x^2+2x+1} \nexists$



$\lim\limits_{x\to -1^-} \dfrac{x^3+x^2}{x^2+2x+1} = \left(\dfrac{0}{0} \text{ IND}\right) = \lim\limits_{x\to -1^-} \dfrac{x^2(x+1)}{(x+1)(x+1)} = \lim\limits_{x\to -1^-} \dfrac{x^2}{x+1} = \dfrac{1}{0^-} = -\infty$

$\lim\limits_{x\to -1^+} \dfrac{x^3+x^2}{x^2+2x+1} = \lim\limits_{x\to -1^+} \dfrac{x^2}{x+1} = \dfrac{1}{0^+} = +\infty$

c) $\lim\limits_{x\to 1} \dfrac{x^4-1}{x-1} = 4$



	1	0	0	0	-1
1		1	1	1	1
	1	1	1	1	$\boxed{0}$
-1		-1	0	-1	
	1	0	1	$\boxed{0}$	

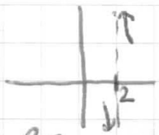
$\lim\limits_{x\to 1^-} \dfrac{x^4-1}{x-1} = \lim\limits_{x\to 1^-} \dfrac{(x+1)(x-1)(x^2+1)}{x-1} = \lim\limits_{x\to 1^-} \dfrac{(x+1)(x^2+1)}{1} = 4$

$x^2+1=0$

$x=\pm\sqrt{-1}$ No té solució real

$\lim\limits_{x\to 1^+} \dfrac{x^4-1}{x-1} = \lim\limits_{x\to 1^+} (x+1)(x^2+1) = 4$

d) $\lim\limits_{x\to 2} \dfrac{2x^2-8}{x^2-4x+4}$ $\nexists$



$\lim\limits_{x\to 2^-} \dfrac{2x^2-8}{x^2-4x+4} = \lim\limits_{x\to 2^-} \dfrac{2(x-2)(x+2)}{(x-2)(x-2)} = \lim\limits_{x\to 2^-} \dfrac{2(x+2)}{(x-2)} = \dfrac{8}{0^-} = -\infty$

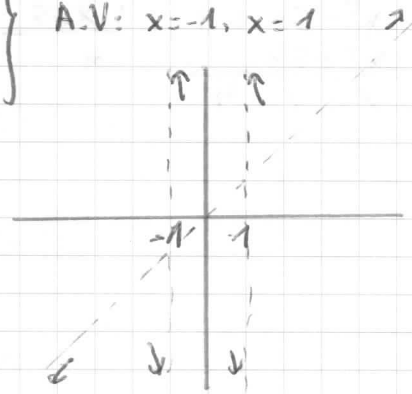
$\lim\limits_{x\to 2^+} \dfrac{2x^2-8}{x^2-4x+4} = \lim\limits_{x\to 2^+} \dfrac{2(x+2)}{(x-2)} = \dfrac{8}{0^+} = +\infty$

54 a) $y = \dfrac{x^3}{x^2-1}$ $\quad x^2-1=0 \to x=\pm 1$

$\lim\limits_{x\to -1^-} \dfrac{x^3}{x^2-1} = \dfrac{-1}{0^+} = -\infty$ $\quad \lim\limits_{x\to -1^+} \dfrac{x^3}{x^2-1} = \dfrac{-1}{0^-} = +\infty$

$\lim\limits_{x\to 1^-} \dfrac{x^3}{x^2-1} = \dfrac{1}{0^-} = -\infty$ $\quad \lim\limits_{x\to 1^+} \dfrac{x^3}{x^2-1} = \dfrac{1}{0^+} = +\infty$

A.V: $x=-1$, $x=1$



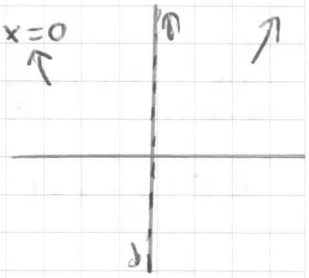
$gr(x^3) - gr(x^2-1) = 3-2 = 1$

$$\begin{array}{l|l} x^3 & x^2-1 \\ \hline -x^3+x & x \\ \hline \quad / \quad x & \end{array} \Rightarrow \dfrac{x^3}{x^2-1} = x + \dfrac{x}{x^2-1}$$

A.O: $y=x$

b) $y = \dfrac{x^3+1}{x}$ $\quad x=0$

$\lim\limits_{x\to 0^-} \dfrac{x^3+1}{x} = \dfrac{1}{0^-} = -\infty$ $\quad \lim\limits_{x\to 0^+} \dfrac{x^3+1}{x} = \dfrac{1}{0^+} = +\infty$ $\Rightarrow$ A.V: $x=0$



$gr(x^3+1) - gr(x) = 3-1 = 2$ $\Rightarrow$ branques parabòliques cap amunt

$\lim\limits_{x\to +\infty} \dfrac{x^3+1}{x} = +\infty$

c) $y = \dfrac{2x^2+5}{x^2-4x+5}$ $\quad x^2-4x+5=0 \to x = \dfrac{-(-4)\pm\sqrt{(-4)^2-4\cdot 1\cdot 5}}{2\cdot 1} = \dfrac{4\pm\sqrt{-4}}{2}$ No té solució real

No té A.V.

$\lim\limits_{x\to +\infty} \dfrac{2x^2+5}{x^2-4x+5} = \dfrac{2}{1} = 2$ $\quad \lim\limits_{x\to -\infty} \dfrac{2x^2+5}{x^2-4x+5} = \dfrac{2}{1} = 2$ $\Rightarrow$ A.H: $y=2$

$\dfrac{2x^2+5}{x^2-4x+5} - 2 = \dfrac{\cancel{2x^2}+5-\cancel{2x^2}+8x-10}{x^2-4x+5} = \dfrac{8x-5}{x^2-4x+5}$

d) $y = \dfrac{x^2+1}{(x^2-1)^2}$ $\quad (x^2-1)^2=0 \to x^2-1=0 \to x=\pm 1$

$\lim\limits_{x\to -1^-} \dfrac{x^2+1}{(x^2-1)^2} = \dfrac{2}{0^+} = +\infty$, $\quad \lim\limits_{x\to -1^+} \dfrac{x^2+1}{(x^2-1)^2} = \dfrac{2}{0^+} = +\infty$

$$\lim_{x\to 1^+} \frac{x^2+1}{(x^2-1)^2} = \frac{2}{0^+} = +\infty \qquad \lim_{x\to 1^+} \frac{x^2+1}{(x^2-1)^2} = \frac{2}{0^+} = +\infty$$ A.V: $x=-1, x=1$

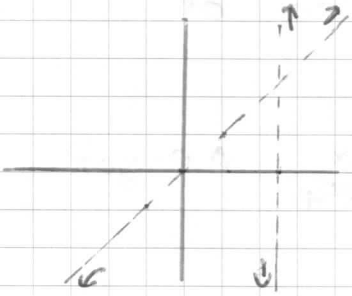
$$\lim_{x\to +\infty} \frac{x^2+1}{(x^2-1)^2} = 0 \qquad \lim_{x\to -\infty} \frac{x^2+1}{(x^2-1)^2} = 0 \Rightarrow \text{AH}: y=0$$

e) $y = \dfrac{x^2-5x+4}{x-5}$ $\qquad x-5=0 \rightarrow x=5$

$$\lim_{x\to 5^-} \frac{x^2-5x+4}{x-5} = \frac{4}{0^-} = -\infty, \quad \lim_{x\to 5^+} \frac{x^2-5x+4}{x-5} = \frac{4}{0^+} = +\infty \Rightarrow \text{A.V}: x=5$$

$gr(x^2-5x+4) - gr(x-5) = 2-1=1$

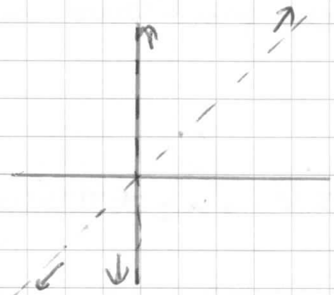
$$\begin{array}{l|l} x^2-5x+4 & x-5 \\ \hline -x^2+5x & x \\ \hline / \quad / \quad 4 & \end{array} \Rightarrow \frac{x^2-5x+4}{x-5} = x + \frac{4}{x-5} \Rightarrow \text{A.O}: y=x$$



f) $y = x+1+\dfrac{5}{x}$

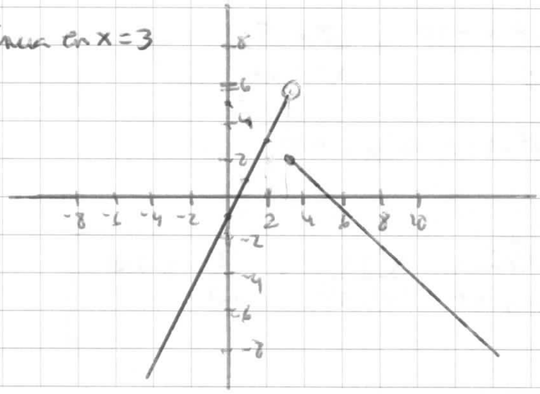
$$\lim_{x\to 0^-} x+1+\frac{5}{x} = -\infty \qquad \lim_{x\to 0^+} x+1+\frac{5}{x} = +\infty \Rightarrow \text{A.V}: x=0$$

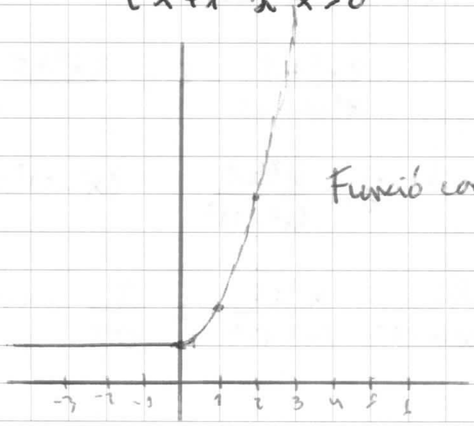
A.O: $y = x+1$



55 a) $f(x) = \begin{cases} 2x-1 & \text{si } x<3 \\ 5-x & \text{si } x\geq 3 \end{cases}$ $\qquad$ b) $f(x) = \begin{cases} 1 & \text{si } x\leq 0 \\ x^2+1 & \text{si } x>0 \end{cases}$

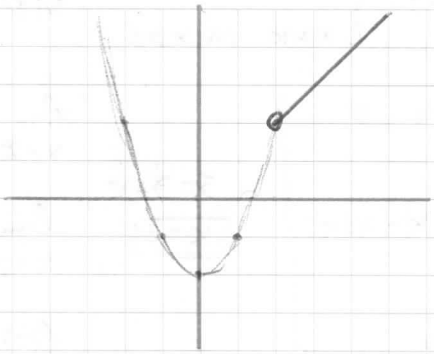
Discontinua en x=3





Funció contínua

c) $f(x) = \begin{cases} x^2-2 & \text{si } x<2 \\ x & \text{si } x>2 \end{cases}$



Discontinua en $x=2$

(56) a) $\lim_{x\to -3^-} 2x-1 = -7$, $\lim_{x\to -3^+} 5-x = 8$ $\Rightarrow \lim_{x\to -3} f(x) \nexists$

$\lim_{x\to 5} 5-x = 0$

$\lim_{x\to +\infty} 5-x = -\infty$ $\qquad \lim_{x\to -\infty} 2x-1 = -\infty$

b) $\lim_{x\to -3} 1 = 1$

$\lim_{x\to 5} x^2+1 = 26$

$\lim_{x\to +\infty} x^2+1 = +\infty$ $\qquad \lim_{x\to -\infty} 1 = 1$

c) $\lim_{x\to -3} x^2-2 = 7$

$\lim_{x\to 5} x = 5$

$\lim_{x\to +\infty} x = +\infty$ $\qquad \lim_{x\to -\infty} x^2-2 = +\infty$

(57) a) $f(x) = 2^{x-1}$ $\qquad$ b) $f(x) = 0'75^x$ $\qquad$ c) $f(x) = 1+e^x$ $\qquad$ d) $f(x) = \frac{1}{e^x}$

$\lim_{x\to +\infty} 2^{x-1} = +\infty$ $\qquad \lim_{x\to +\infty} 0'75^x = 0$ $\qquad \lim_{x\to +\infty} 1+e^x = +\infty$ $\qquad \lim_{x\to +\infty} \frac{1}{e^x} = 0$

$\lim_{x\to -\infty} 2^{x-1} = 0$ $\qquad \lim_{x\to -\infty} 0'75^x = +\infty$ $\qquad \lim_{x\to -\infty} 1+e^x = 1$ $\qquad \lim_{x\to -\infty} \frac{1}{e^x} = +\infty$

(58) a) $y = 2^{x+3}$ $\qquad$ b) $y = 1'5^x - 1$ $\qquad$ c) $y = 2+e^x$ $\qquad$ d) $y = e^{-x}$

$\lim_{x\to +\infty} 2^{x+3} = +\infty$ $\qquad \lim_{x\to +\infty} 1'5^x - 1 = +\infty$ $\qquad \lim_{x\to +\infty} 2+e^x = +\infty$ $\qquad \lim_{x\to +\infty} e^{-x} = 0$

$\lim_{x\to -\infty} 2^{x+3} = 0$ $\qquad \lim_{x\to -\infty} 1'5^x - 1 = -1$ $\qquad \lim_{x\to -\infty} 2+e^x = 2$ $\qquad \lim_{x\to -\infty} e^{-x} = +\infty$

59) a) $f(x)=\begin{cases} x^2-4 & \text{si } x\le 3 \\ x+k & \text{si } x>3 \end{cases}$

$\lim_{x\to 3^-} x^2-4 = 5$

$\lim_{x\to 3^+} x+k = 3+k$

$f(3)=3+k$

$\Rightarrow 5=3+k$ $\boxed{k=2}$

b) $f(x)=\begin{cases} 6-\frac{x}{2} & \text{si } x<2 \\ x^2+kx & \text{si } x\ge 2 \end{cases}$

$\lim_{x\to 2^-} 6-\frac{x}{2} = 5$

$\lim_{x\to 2^+} x^2+kx = 4+2k$

$f(2)=2^2+2k=4+2k$

$4+2k=5$

$2k=1$

$\boxed{k=\frac{1}{2}}$

c) $f(x)=\begin{cases} \frac{x^2+x}{x} & \text{si } x\ne 0 \\ k & \text{si } x=0 \end{cases}$

$\lim_{x\to 0} \frac{x^2+x}{x} = \left(\frac{0}{0} \text{ IND}\right) =$

$\lim_{x\to 0} \frac{x(x+1)}{x} = \lim_{x\to 0} x+1 = 1$

$f(0)=k$

$\Rightarrow \boxed{k=1}$

60) a) $f(x)=\begin{cases} 2-x & \text{si } x<1 \\ \frac{1}{x} & \text{si } x\ge 1 \end{cases}$

$\lim_{x\to 1^-} 2-x = 1$

$\lim_{x\to 1^+} \frac{1}{x} = 1$

$f(1)=\frac{1}{1}=1$

Com $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

és una funció contínua en $x=1$

Si $x\ne 1$ la funció és contínua

→ La funció és contínua en $\mathbb{R}$

b) $f(x)=\begin{cases} -x-1 & \text{si } x\le -1 \\ 1-x^2 & \text{si } -1<x<1 \\ x-1 & \text{si } x\ge 1 \end{cases}$

$\lim_{x\to -1^-} -x-1 = 0$

$\lim_{x\to -1^+} 1-x^2 = 0$

$f(-1)=-(-1)-1=0$

la funció és contínua en $x=-1$

$\lim_{x\to 1^-} 1-x^2 = 0$

$\lim_{x\to 1^+} x-1 = 0$

$f(1)=1-1=0$

la funció és contínua en $x=1$

Si $x\ne 1$ i $x\ne -1$ la funció és contínua.

→ La funció és contínua en $\mathbb{R}$

b) $f(x)=\begin{cases} 1-x^2 & \text{si } x\le 0 \\ 2^{x+1} & \text{si } x>0 \end{cases}$

$\lim_{x\to 0^-} 1-x^2 = 1$

$\lim_{x\to 0^+} 2^{x+1} = 2$

$f(0)=1-0^2=1$

Discontínua en $x=0$

→ Si $x\ne 0$, la funció és contínua.

61) a) $f(x)=\begin{cases} x+1 & \text{si } x\le 1 \\ 4-ax^2 & \text{si } x>1 \end{cases}$

$\lim_{x\to 1^-} x+1 = 2$

$\lim_{x\to 1^+} 4-ax^2 = 4-a$

$f(1)=1+1=2$

$4-a=2$

$\boxed{2=a}$

b) $f(x)=\begin{cases} \frac{x^2-1}{x-1} & \text{si } x\ne 1 \\ a & \text{si } x=1 \end{cases}$

$\lim_{x\to 1} \frac{x^2-1}{x-1} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)} =$

$= \lim_{x\to 1} x+1 = 2$

$f(1)=a$

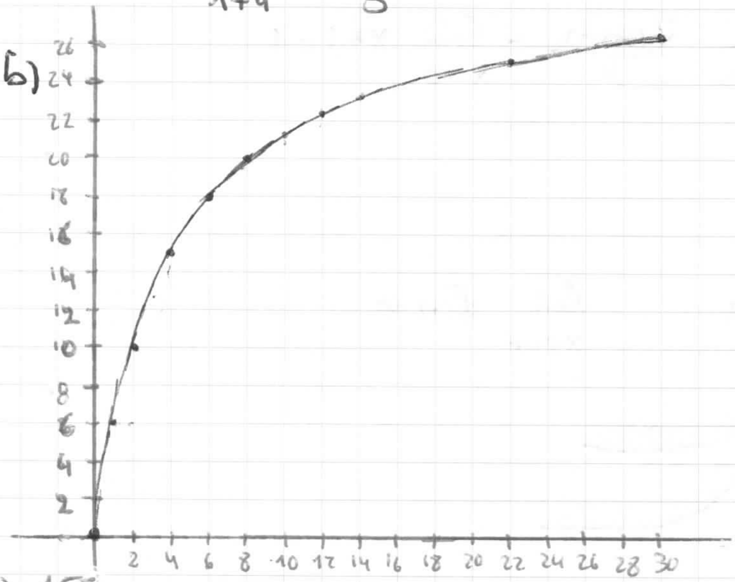
$\Rightarrow \boxed{a=2}$

(62) $M(t) = \frac{30t}{t+4}$ (t en dies) $\quad M(t) = 30 + \frac{-120}{t+4}$

$$\begin{array}{r|l} 30t & t+4 \\ -30t-120 & 30 \\ \hline / \quad -120 & \end{array}$$

a) $M(1) = \frac{30 \cdot 1}{1+4} = \frac{30}{5} = 6$. El primer dia farà 6 muntatges.

b)



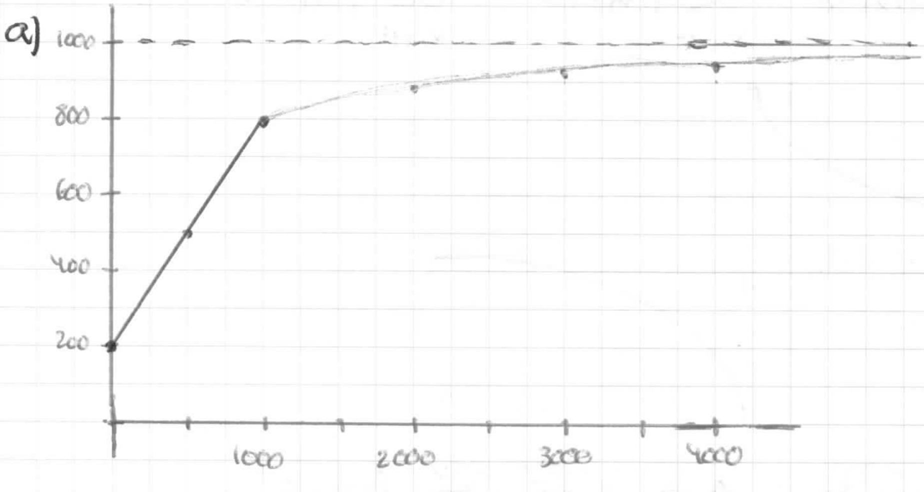
c) $\lim_{x \to +\infty} \frac{30t}{t+4} = 30$

S'apropa a 30 muntatges.

Pàg 155

(63) $g(x) = \begin{cases} 0'6x + 200 & \text{si } 0 \leq x \leq 1000 \\ \frac{1000x}{x+250} & \text{si } x > 1000 \end{cases}$

a)



És contínua

b) $\lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} \frac{1000x}{x+250} = 1000$

Com a màxim gasten 1000€ al mes

Qüestions tècniques

(64) Sí que es pot calcular el límit d'una funció en un punt en el qual la funció no estigui definida.

No pot ser contínua en aquest punt.

(65) Sí, per exemple $f(x) = \dfrac{1}{x^2-x-2}$ té $x=1$ i $x=-2$ com A.V.

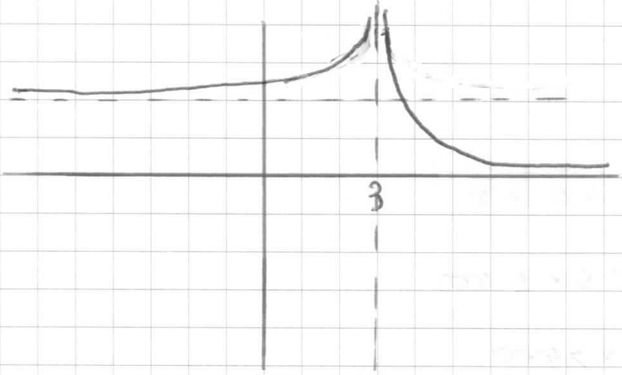
$(x+1)(x-2) = x^2-2x+x-2 = x^2-x-2$

(66) No, per exemple $f(x) = \dfrac{x^2+x}{x}$ en $x=0$ ja que

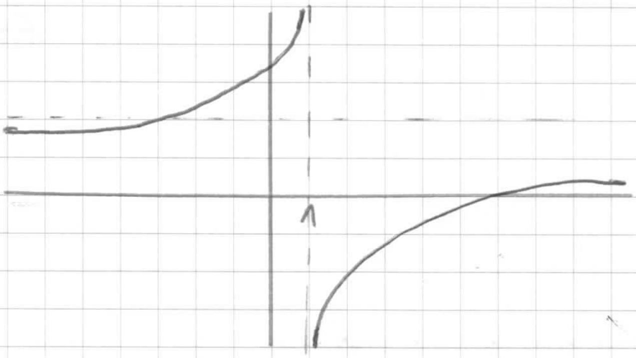
$$\lim_{x\to 0} \frac{x^2+x}{x} = \left(\frac{0}{0} \text{ IND}\right) = \lim_{x\to 0} \frac{x(x+1)}{x} = \lim_{x\to 0} x+1 = 1$$

(67) Sí.

(68) $\lim\limits_{x\to 3} f(x) = +\infty$ $\quad \lim\limits_{x\to -\infty} f(x) = 2$ $\quad \lim\limits_{x\to +\infty} f(x) = 0$



(69) $\lim\limits_{x\to -\infty} f(x) = 2$ $\quad \lim\limits_{x\to +\infty} f(x) = 0$ $\quad \lim\limits_{x\to 1^-} f(x) = +\infty$ $\quad \lim\limits_{x\to 1^+} f(x) = -\infty$



(70) No, perquè si fos contínua hauria de ser, a més a més, $f(2) = 5$.

(71) $f(x) = \begin{cases} \dfrac{1}{x} & \text{si } x \neq 0 \\ k & \text{si } x = 0 \end{cases}$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{1}{x} = \dfrac{1}{0^-} = -\infty \\ \lim\limits_{x\to 0^+} \dfrac{1}{x} = \dfrac{1}{0^+} = +\infty \end{array}\right\} \lim_{x\to 0} f(x) \text{ no existeix}$$

$f(0) = k$

No existeix cap valor de k perquè la funció sigui contínua.

Per aprofundir

(72) a) $\lim_{x\to+\infty} \sqrt{\frac{x+3}{x-2}} = \sqrt{\frac{1}{1}} = 1$

c) $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to-\infty} \frac{|x|}{x} = -1$

b) $\lim_{x\to+\infty} \frac{\sqrt{x+1}}{x} = 0$

d) $\lim_{x\to+\infty} \frac{3x-1}{\sqrt{x^2+4}} = \frac{3}{\sqrt{1}} = 3$

(73) $x = 100 \rightarrow 100^2 - 3\cdot 100 = 10000 - 300 = 9700.$

(74) $x = 1000 \rightarrow \frac{1}{3\cdot 1000^{5}} < 0'00034$

(75) a) $\lim_{x\to+\infty} (\sqrt{x} - x) = -\infty$

c) $\lim_{x\to+\infty} \frac{x}{e^x} = 0$

b) $\lim_{x\to+\infty} (2^x - x^3) = +\infty$

d) $\lim_{x\to-\infty} (0'75^x - x) = +\infty$

(76) a) $y = \log_2(x-3)$

$x-3>0 \rightarrow x>3$

A.V: $x=3$

$\lim_{x\to+\infty} \log_2(x-3) = +\infty$

b) $y = \ln(x+2)$

$x+2>0 \rightarrow x>-2$

A.V: $x=-2$

$\lim_{x\to+\infty} \ln(x+2) = +\infty$

Per pensar una mica més.

(77) a)

VELOCITAT BAIXADA	60	80	100	200
VELOCITAT PUJADA	30	32	$33'\hat{3}$	$36'\widehat{36}$

b) d = distància (Km) en la pujada ⇒ 2d = distància (Km) en la pujada i baixada.

$$\text{Velocitat} = \frac{\text{espai}}{\text{temps}}$$

$$\text{temps: } \underbrace{\frac{d}{20}}_{\text{pujada}} + \underbrace{\frac{d}{x}}_{\text{baixada}} = \frac{xd + 20d}{20x} = d\left(\frac{x+20}{20x}\right) \rightarrow T(x) = d\left(\frac{x+20}{20x}\right)$$

$$V_{pf}(x) = \frac{2d}{d\left(\frac{x+20}{20x}\right)} \rightarrow V_{pf}(x) = \frac{2}{\frac{x+20}{20x}} \rightarrow \boxed{V_{pf}(x) = \frac{40x}{x+20}}$$

c) $\lim_{x \to +\infty} V_F(x) = \lim_{x \to +\infty} \frac{40x}{x+20} = 40 \ Km/h$