Pàg 12. Reflexiona i resol

a) $3x = 15 \to x = \frac{15}{3} \to x = 5 \in \mathbb{Z}$ b) $-2x = 18 \to x = \frac{18}{-2} \to x = -9 \in \mathbb{Z}$

c) $11x = -341 \to x = \frac{-341}{11} \to x = -31 \in \mathbb{Z}$ d) $4x = 34 \to x = \frac{34}{4} \to x = \frac{17}{2} \in \mathbb{Q}$

a) $-5x = 60 \to x = \frac{60}{-5} \to x = -12 \in \mathbb{Z}$ b) $-7x = 22 \to x = \frac{22}{-7} \in \mathbb{Q}$

c) $2x + 1 = 15 \to 2x = 14 \to x = 7 \in \mathbb{Z}$ d) $6x - 2 = 10 \to x = \frac{12}{6} \to x = \in \mathbb{Z}$

e) $-3x - 3 = 1 \to x = \frac{4}{-3} \in \mathbb{Q}$ f) $-x + 7 = 6 \to x = \frac{-1}{-1} \to x = 1 \in \mathbb{Z}$

Pàg 13.

a) $3x^2 - 12 = 0 \to x^2 - 4 = 0 \to x^2 = 4 \to x = \pm\sqrt{4} \to x = \pm 2 \in \mathbb{Q}$

b) $x^2 - 6x + 8 = 0 \to x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2} = \begin{cases} x_1 = 4 \in \mathbb{Q} \\ x_2 = 2 \in \mathbb{Q} \end{cases}$

c) $2x^2 + x - 1 = 0 \to x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4} = \begin{cases} x_1 = \frac{2}{4} = \frac{1}{2} \in \mathbb{Q} \\ x_2 = \frac{-4}{4} = -1 \in \mathbb{Q} \end{cases}$

d) $x^2 - 2 = 0 \to x^2 = 2 \to x = \pm\sqrt{2} \begin{cases} x = -\sqrt{2} \in \mathbb{R} \\ x = \sqrt{2} \in \mathbb{R} \end{cases}$

a) $x^2 - 9 = 0 \to x^2 = 9 \to x = \pm\sqrt{9} \to x = \pm 3 \begin{cases} x_1 = -3 \in \mathbb{Q} \\ x_2 = 3 \in \mathbb{Q} \end{cases}$

b) $5x^2 - 15 = 0 \to x^2 - 3 = 0 \to x^2 = 3 \to x = \pm\sqrt{3} \begin{cases} x_1 = -\sqrt{3} \in \mathbb{R} \\ x_2 = \sqrt{3} \in \mathbb{R} \end{cases}$

c) $x^2 - 3x - 4 = 0 \to x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2} = \begin{cases} x_1 = \frac{8}{2} = 4 \in \mathbb{Q} \\ x_2 = \frac{-2}{2} = -1 \in \mathbb{Q} \end{cases}$

d) $2x^2 - 5x + 1 = 0 \to x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{5 \pm \sqrt{17}}{4} = \begin{cases} x_1 = \frac{5 - \sqrt{17}}{4} \in \mathbb{R} \\ x_2 = \frac{5 + \sqrt{17}}{4} \in \mathbb{R} \end{cases}$

e) $7x^2 - 7x = 0 \to x^2 - x = 0 \to x(x-1) = 0 \begin{cases} x_1 = 0 \in \mathbb{Q} \\ (x-1) = 0 \to x = 1 \in \mathbb{Q} \end{cases}$

f) $2x^2 + 3x = 0 \to x(2x+3) = 0 \begin{cases} x_1 = 0 \in \mathbb{Q} \\ 2x + 3 = 0 \to x = \frac{-3}{2} \in \mathbb{Q} \end{cases}$

a) $x^2-2=0 \rightarrow x^2=2 \rightarrow x=\pm\sqrt{2}$ $\begin{cases} x_1=-\sqrt{2} \in \mathbb{R} \\ x_2=\sqrt{2} \in \mathbb{R} \end{cases}$

b) $2x^2-5x+1=0 \rightarrow x=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot2\cdot1}}{2\cdot2}=\frac{5\pm\sqrt{17}}{4}=\begin{cases} x_1=\frac{5-\sqrt{17}}{4}\in\mathbb{R} \\ x_2=\frac{5+\sqrt{17}}{4}\in\mathbb{R} \end{cases}$

c) $5x^2-x-2=0 \rightarrow x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot5\cdot(-2)}}{2\cdot5}=\frac{1\pm\sqrt{41}}{10}=\begin{cases} x_1=\frac{1-\sqrt{41}}{10}\in\mathbb{R} \\ x_2=\frac{1+\sqrt{41}}{10}\in\mathbb{R} \end{cases}$

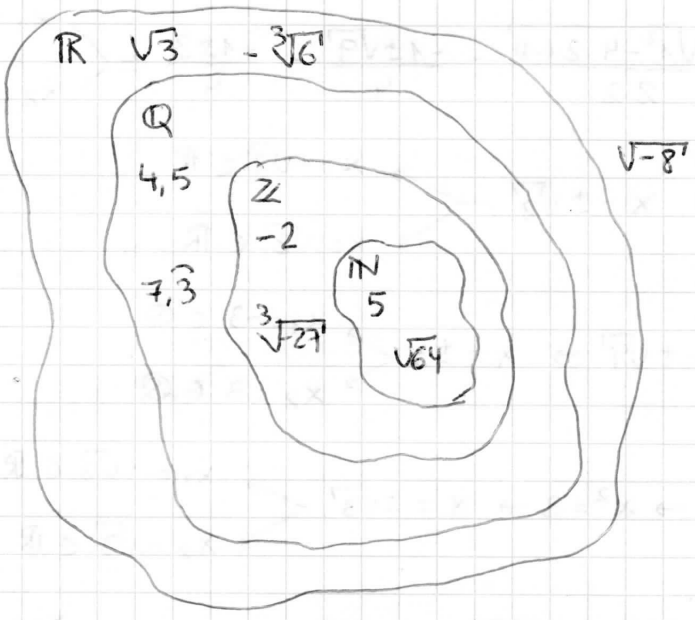
d) $x^2+1=0 \rightarrow x^2=-1 \rightarrow x=\pm\sqrt{-1}$ $\begin{cases} x_1=-i\in\mathbb{C} \\ x_2=i\in\mathbb{C} \end{cases}$

e) $x^2-2x+5=0 \rightarrow x=\frac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot5}}{2\cdot1}=\frac{2\pm\sqrt{-16}}{2}=1\pm\sqrt{-4}$ $\begin{cases} x_1=1-2i \\ x_2=1+2i \end{cases}$ ($\mathbb{C}$)

f) $5x^2+10=0 \rightarrow x^2+2=0 \rightarrow x^2=-2 \rightarrow x=\pm\sqrt{-2}$ $\begin{cases} x_1=-\sqrt{2}i\in\mathbb{C} \\ x_2=\sqrt{2}i\in\mathbb{C} \end{cases}$ ($\mathbb{C}$)

Pàg 14

1.



2. ℕ: 5, $\sqrt{64}$

ℤ: 5, $\sqrt{64}$, -2, $\sqrt[3]{-27}$

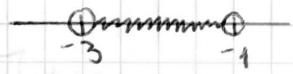
ℚ: 5, $\sqrt{64}$, -2, $\sqrt[3]{-27}$, 4'5, 7'3̂

ℝ: 5, $\sqrt{64}$, -2, $\sqrt[3]{-27}$, 4'5, 7'3̂, $\sqrt{3}$, $-\sqrt[3]{6}$

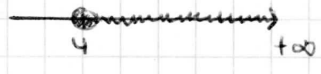
No reals: $\sqrt{-8}$

Pàg 15

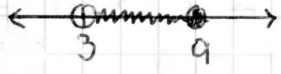
3. a) $(-3,-1)=\{x\in\mathbb{R} \mid -3<x<1\}$



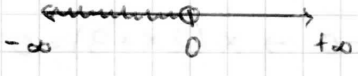
b) $[4,+\infty)=\{x\in\mathbb{R} \mid 4\leq x\}$



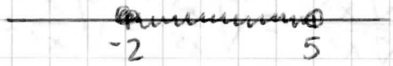
c) $(3,9]=\{x\in\mathbb{R} \mid 3<x\leq 9\}$



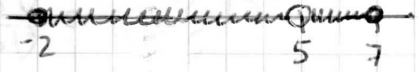
d) $(-\infty,0)=\{x\in\mathbb{R} \mid x<0\}$



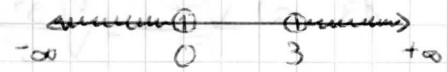
4. a) $\{x \in \mathbb{R} \mid -2 \leq x < 5\} = [-2, 5)$



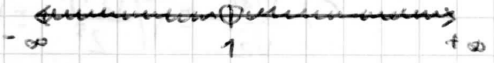
b) $[-2,5) \cup (5,7] = \{x \in \mathbb{R} \mid -2 \leq x < 5\} \cup \{x \in \mathbb{R} \mid 5 < x \leq 7\}$



c) $(-\infty, 0) \cup (3, +\infty) = \{x \in \mathbb{R} \mid x < 0\} \cup \{x \in \mathbb{R} \mid 3 < x\}$



d) $(-\infty, 1) \cup (1, +\infty) = \{x \in \mathbb{R} \mid x < 1\} \cup \{x \in \mathbb{R} \mid 1 < x\}$



Pàg 16

5. a) $|-11| = 11$ b) $|\pi| = \pi$ c) $|-\sqrt{5}| = \sqrt{5}$ d) $|0| = 0$

e) $|3-\pi| = \pi - 3$ f) $|3-\sqrt{2}| = 3-\sqrt{2}$ g) $|1-\sqrt{2}| = \sqrt{2}-1$ h) $|\sqrt{2}-\sqrt{3}| = \sqrt{3}-\sqrt{2}$

i) $|7-\sqrt{50}| = \sqrt{50}-7$

6. a) $|x| = 5 \rightarrow x = -5$ o $x = 5$

b) $|x| \leq 5 \rightarrow -5 \leq x \leq 5 \rightarrow x \in [-5,5]$

c) $|x-4| = 2 \rightarrow x-4 = 2 \rightarrow x = 6$ o $x-4 = -2 \rightarrow x = 2$

d) $|x-4| > 2 \rightarrow x-4 > 2 \rightarrow x > 6$ o $x-4 < -2 \rightarrow x < 2$

e) $|x-4| \leq 2 \rightarrow -2 \leq x-4 \leq 2 \rightarrow 2 \leq x \leq 6 \rightarrow [2,6]$

f) $|x+4| > 5 \rightarrow x+4 > 5 \rightarrow x > 1$ o $x+4 < -5 \rightarrow x < -9$

Pàg 17

7. a) $\sqrt[12]{x^9} = \sqrt[4]{x^3}$ b) $\sqrt[12]{x^8} = \sqrt[3]{x^2}$ c) $\sqrt[5]{y^{10}} = y^2$

d) $\sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt{2}$ e) $\sqrt[9]{64} = \sqrt[9]{2^6} = \sqrt[3]{2^2}$ f) $\sqrt[8]{81} = \sqrt[8]{3^4} = \sqrt{3}$

8. $\sqrt[4]{31} > \sqrt[3]{13}$ $\sqrt[4]{31} = \sqrt[12]{31^3} = \sqrt[12]{29791}$ $\sqrt[12]{13^4} = \sqrt[12]{28561}$

9. a) $\sqrt[12]{a^5} = \sqrt[36]{a^{15}}$ $\sqrt[18]{a^7} = \sqrt[36]{a^{14}}$

b) $\sqrt[3]{51} = \sqrt[9]{51^3} = \sqrt[9]{132651}$ $\sqrt[9]{132650}$

10. a) $(\sqrt{\sqrt{\sqrt{k}}})^8 = (\sqrt[8]{k})^8 = k$ b) $\sqrt[5]{\sqrt[3]{x^{10}}} = \sqrt[15]{x^{10}} = \sqrt[3]{x^2}$ c) $\sqrt[3]{(\sqrt{x})^6} = \sqrt[6]{x^6} = x$

Pàg 18

11. a) $\sqrt[3]{2} \cdot \sqrt[5]{2} = \sqrt[15]{2^5} \cdot \sqrt[15]{2^3} = \sqrt[15]{2^8}$ b) $\sqrt[3]{9} \cdot \sqrt[6]{3} = \sqrt[6]{(3^2)^2} \cdot \sqrt[6]{3} = \sqrt[6]{3^5}$

c) $\sqrt{2} \cdot \sqrt[4]{2} \cdot \sqrt[8]{2} = \sqrt[8]{2^4} \cdot \sqrt[8]{2^2} \cdot \sqrt[8]{2} = \sqrt[8]{2^7}$ d) $\sqrt[4]{8} \cdot \sqrt[3]{4} = \sqrt[12]{(2^3)^3} \cdot \sqrt[12]{(2^2)^4} = \sqrt[12]{2^{17}}$

12. a) $\frac{\sqrt[5]{x}}{\sqrt[3]{x}} = \frac{\sqrt[15]{x^3}}{\sqrt[15]{x^5}} = \sqrt[15]{\frac{x^3}{x^5}} = \sqrt[15]{\frac{1}{x^2}} = \sqrt[15]{x^{-2}}$ b) $\frac{\sqrt{a \cdot b}}{\sqrt[3]{a \cdot b}} = \frac{\sqrt[6]{a^3 b^3}}{\sqrt[6]{a^2 \cdot b^2}} = \sqrt[6]{\frac{a^3 b^3}{a^2 b^2}} = \sqrt[6]{ab}$

c) $\frac{\sqrt[6]{a^3}}{\sqrt[3]{a^2}} = \frac{\sqrt[6]{a^3}}{\sqrt[6]{a^4}} = \sqrt[6]{\frac{a^3}{a^4}} = \sqrt[6]{\frac{1}{a}} = \sqrt[6]{a^{-1}}$ d) $\frac{\sqrt[4]{a^3 b^5 c}}{\sqrt{ab^3 c^3}} = \frac{\sqrt[4]{a^3 b^5 c}}{\sqrt[4]{a^2 b^6 c^6}} = \sqrt[4]{\frac{a^3 b^5 c}{a^2 b^6 c^6}} = \sqrt[4]{\frac{a}{bc^5}} = \frac{1}{c}\sqrt[4]{\frac{a}{bc}}$

13. a) $\frac{\sqrt[3]{3^2}}{\sqrt{3}} = \sqrt[6]{\frac{3^4}{3^3}} = \sqrt[6]{3}$ b) $\frac{\sqrt{9}}{\sqrt[3]{3}} = \sqrt[6]{\frac{(3^2)^3}{3^2}} = \sqrt[6]{\frac{3^6}{3^2}} = \sqrt[6]{3^4} = \sqrt[3]{3^2}$

c) $\frac{\sqrt[5]{16}}{\sqrt{2}} = \sqrt[10]{\frac{(2^4)^2}{2^5}} = \sqrt[10]{\frac{2^8}{2^5}} = \sqrt[10]{2^3} = \sqrt[10]{8}$ d) $\frac{\sqrt[4]{729}}{\sqrt{3}} = \sqrt[4]{\frac{3^6}{3^2}} = \sqrt[4]{3^4} = 3$

14. a) $5\sqrt{x} + 3\sqrt{x} + 2\sqrt{x} = 10\sqrt{x}$

b) $\sqrt{9 \cdot 2} + \sqrt{25 \cdot 2}\ \sqrt{2} = 3\sqrt{2} + 5\sqrt{2} - \sqrt{2} = 7\sqrt{2}$

c) $\sqrt{18} + \sqrt{50} - \sqrt{2} - \sqrt{8} = \sqrt{3^2 \cdot 2} + \sqrt{5^2 \cdot 2} - \sqrt{2} - \sqrt{2^3} = 3\sqrt{2} + 5\sqrt{2} - \sqrt{2} - 2\sqrt{2} =$

$= 5\sqrt{2}$

d) $\sqrt{27} - \sqrt{50} + \sqrt{12} + \sqrt{8} = \sqrt{3^3} - \sqrt{5^2 \cdot 2} + \sqrt{2^2 \cdot 3} + \sqrt{2^3} = 3\sqrt{3} - 5\sqrt{2} + 2\sqrt{3} + 2\sqrt{2} =$

$= 5\sqrt{3} - 3\sqrt{2}$

e) $\sqrt{50a} - \sqrt{18a} = \sqrt{5^2 \cdot 2 \cdot a} - \sqrt{3^2 \cdot 2 \cdot a} = 5\sqrt{2a} - 3\sqrt{2a} = 2\sqrt{2a}$

Pàg 19

15. a) $\frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$ b) $\frac{3}{\sqrt[3]{4}} = \frac{3}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{3\sqrt[3]{2}}{2}$

c) $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$ d) $\frac{1}{\sqrt{a^3}} = \frac{1}{a\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a \cdot a} = \frac{\sqrt{a}}{a^2}$

e) $\frac{3}{\sqrt{50}} = \frac{3}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{10}$ f) $\frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

g) $\frac{2}{\sqrt[3]{25}} = \frac{2}{\sqrt[3]{5^2}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{2\sqrt[3]{5}}{5}$ h) $\frac{1}{\sqrt[3]{40}} = \frac{1}{2\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} = \frac{\sqrt[3]{5^2}}{2 \cdot 5} = \frac{\sqrt[3]{25}}{10}$

i) $\frac{3}{\sqrt[3]{36}} = \frac{3}{\sqrt[3]{6^2}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}} = \frac{3\sqrt[3]{6}}{6} = \frac{\sqrt[3]{6}}{2}$ j) $\frac{2}{\sqrt[3]{100}} = \frac{2}{\sqrt[3]{10^2}} \cdot \frac{\sqrt[3]{10}}{\sqrt[3]{10}} = \frac{2\sqrt[3]{10}}{10} = \frac{\sqrt[3]{10}}{5}$

16. a) $\frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$ b) $\frac{x+y}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{x\sqrt{x} - x\sqrt{y} + y\sqrt{x} - y\sqrt{y}}{x-y}$

c) $\frac{a-1}{\sqrt{a}-1} \cdot \frac{\sqrt{a}+1}{\sqrt{a}+1} = \frac{(a-1)(\sqrt{a}+1)}{(a-1)} = \sqrt{a}+1$ d) $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{(\sqrt{x}+\sqrt{y})^2}{x-y} = \frac{x+y+2\sqrt{xy}}{x-y}$

e) $\frac{1}{2\sqrt{3}-\sqrt{5}} \cdot \frac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}} = \frac{2\sqrt{3}+\sqrt{5}}{12-5} = \frac{2\sqrt{3}+\sqrt{5}}{7}$ f) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \cdot \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{(3\sqrt{2}+2\sqrt{3})^2}{18-12} =$

$= \frac{18+12+12\sqrt{6}}{6} = \frac{30+12\sqrt{6}}{6} = 5+2\sqrt{6}$

g) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}}{2} + \sqrt{2}+1+\sqrt{2}-1 = \frac{\sqrt{2}}{2} + 2\sqrt{2} = \frac{\sqrt{2}}{2} + \frac{4\sqrt{2}}{2} = \frac{5\sqrt{2}}{2}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{1}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$

$\frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

h) $\frac{1}{\sqrt{x}-\sqrt{y}} + \frac{1}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{x-y} + \frac{\sqrt{x}-\sqrt{y}}{x-y} = \frac{\sqrt{x}+\sqrt{y}+\sqrt{x}-\sqrt{y}}{x-y} = \frac{2\sqrt{x}}{x-y}$

$\frac{1}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{x-y}$

$\frac{1}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$

Pàg 22.

17. $\pi \to 3'14$

$\varepsilon_a = |\pi - 3'14| < 0,0016 \qquad \varepsilon_r = \frac{0'0016}{\pi} < 0'00051$

18. $\Phi \to 1'6180339\ldots 100$ $\tilde{\Phi} = 161,8033999$

$\varepsilon_a = |\Phi - 1'6180339| < 0,00000002 \qquad \varepsilon_r = \frac{0'00000002}{\Phi} < 0,000000\bullet002$

19. $d(\text{Terra, Sol}) = 149.000.000\,\text{Km}$

a) $1,49 \cdot 10^8$ Km b) $1,49 \cdot 10^{13} \to 1'5 \cdot 10^{13}$ cm $\quad \varepsilon_r = \frac{1000000}{149000000} < 0,007$

c) $\varepsilon_a = |1'49 \cdot 10^8 - 1'5 \cdot 10^8| = 0,01 \cdot 10^8 = 1000000$ ↑

Pàg 25

20. a) $\log_2 16 \qquad 2^x = 16 \to 2^x = 2^4 \to x = 4$

b) $\log_2 0,25 \qquad 2^x = 0,25 \to 2^x = \frac{1}{4} \to 2^x = \left(\frac{1}{2}\right)^2 \to 2^x = 2^{-2} \to x = -2$

c) $\log_9 1 \qquad 9^x = 1 \to 9^x = 9^0 \to x = 0$

d) $\log_{10} 0'1 \qquad 10^x = 0'1 \to 10^x = 10^{-1} \to x = -1$

e) $\log_4 64 \qquad 4^x = 4^3 \to x = 3$

f) $\log_7 49 \qquad 7^x = 49 \to 7^x = 7^2 \to x = 2$

g) $\ln e^4 \qquad e^x = e^4 \to x = 4$

h) $\ln e^{-1/4} \qquad e^x = e^{-1/4} \to x = -\frac{1}{4}$

i) $\log_5 0'04$ $\quad 5^x = \frac{1}{25} \to 5^x = \left(\frac{1}{5}\right)^2 \to 5^x = 5^{-2} \to x = -2$

j) $\log_6\left(\frac{1}{216}\right)$ $\quad 6^x = \left(\frac{1}{6}\right)^3 \to 6^x = 6^{-3} \to x = -3$

21. a) $\log_2 60$ $\quad 2^x = 60 \to x = 5$ $\quad$ d) $\log_{10} 0{,}084$ $\quad 10^x = 0{,}084$ $\quad x = -1$

b) $\log_5 700$ $\quad 5^x = 700 \to x = 4$ $\quad$ e) $\log_9 60$ $\quad 9^x = 60$ $\quad x = 1$

c) $\log_{10} 43000$ $\quad 10^x = 43000 \to x = 4$ $\quad$ f) $\ln e$ $\quad e^x = e$ $\quad x = 1$

22. a) $\log_2 1500 = \frac{\log 1500}{\log 2} \simeq 10{,}55$ $\quad$ b) $\log_5 200 = \frac{\log 200}{\log 5} \simeq 3{,}29$

c) $\log_{100} 200 = \frac{\log 200}{\log 100} \simeq 1{,}15$ $\quad$ d) $\log_{100} 40 = \frac{\log 40}{\log 100} \simeq 0{,}80$

23. $\log_5 A = 1{,}8$ i $\log_5 B = 2{,}4$

a) $\log_5 \sqrt[3]{\frac{A^2}{25B}} = \frac{\log_5\left[\frac{A^2}{25B}\right]}{3} = \frac{\log_5 A^2 - \log_5(25 \cdot B)}{3} = \frac{2\log_5 A - [\log_5 25 + \log_5 B]}{3} =$

$= \frac{2\log_5 A - [\log_5 5^2 + \log_5 B]}{3} = \frac{2\log_5 A - [2\log_5 5 + \log_5 B]}{3} =$

$= \frac{2 \cdot 1'8 - [2 \cdot 1 + 2{,}4]}{3} = \frac{3'6 - [4'4]}{3} = \frac{-0'8}{3} \simeq -0{,}27$

b) $\log_5\left(\frac{5\sqrt{A^3}}{B^2}\right) = -\log_5(B^2) + \log_5(5\sqrt{A^3}) = -2\log_5 B + [\log_5 5 + \log_5 \sqrt{A^3}] =$

$= 2\log_5 B + \left[1 + \frac{3\log_5 A}{2}\right] = -2 \cdot 2{,}4 + \left[1 + \frac{3 \cdot 1'8}{2}\right] = -4{,}8 + [1 + 2'7]$

$= -4{,}8 + 3'7 = -1'1$

24. $\ln y = 2x - \ln 5 \to y = e^{2x - \ln 5} \to y = e^{2x} \cdot e^{-\ln 5} \to y = \frac{e^{2x}}{e^{\ln 5}} \to y = \frac{e^{2x}}{5}$

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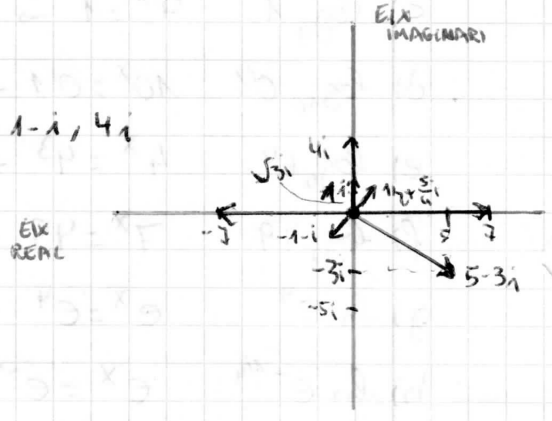
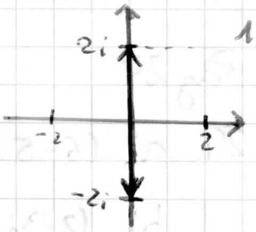
25. Reals: $7, 0, -7$

Imaginaris: $5-3i$, $\frac{1}{2} + \frac{5}{4}i$, $-5i$, $\sqrt{3}i$, $-1-i$, $4i$

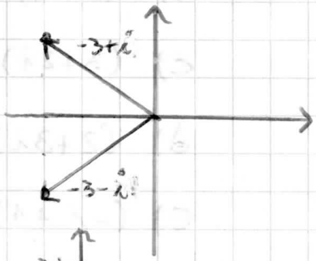
Imaginaris purs: $-5i$, $\sqrt{3}i$, $4i$



26. a) $x^2+4=0 \to x^2=-4 \to x=\pm\sqrt{-4} \to x_1=2i \,,\; x_2=-2i$



b) $x^2+6x+10=0 \to x=\dfrac{-6\pm\sqrt{6^2-4\cdot1\cdot10}}{2\cdot1}=\dfrac{-6\pm\sqrt{-4}}{2}=\begin{cases} x_1=-3-i \\ x_2=-3+i \end{cases}$

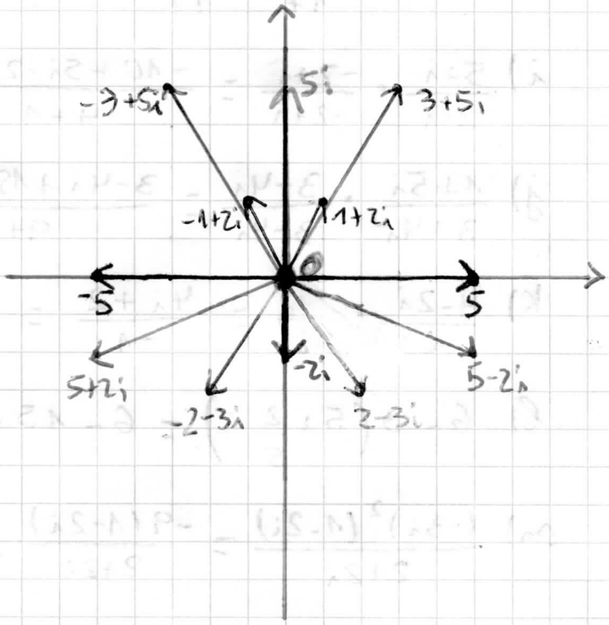


c) $3x^2+27=0 \to x^2+9=0 \to x^2=-9 \to x=\pm\sqrt{9} \begin{cases} x_1=3i \\ x_2=-3i \end{cases}$

d) $3x^2-27=0 \to x^2-9=0 \to x^2=9 \to x=\pm\sqrt{9} \begin{cases} x_1=3 \\ x_2=-3 \end{cases}$

27.

	z	Oposat $(-z)$	Conjugat $(\bar{z})$
a)	$3-5i$	$-3+5i$	$3+5i$
b)	$5+2i$	$-5-2i$	$+5-2i$
c)	$-1-2i$	$1+2i$	$-1+2i$
d)	$-2+3i$	$2-3i$	$-2-3i$
e)	5	-5	5
f)	0	0	0
g)	$2i$	$-2i$	$-2i$
h)	$-5i$	$5i$	$5i$



28. $i^2=-1 \quad i^3=-i \quad i^4=1 \quad i^5=i \quad i^6=-1 \quad i^{20}=i^0=1$

$i^{21}=i^1=i \quad i^{22}=i^2=-1 \quad i^{23}=i^3=-i$

$20 \,\underline{|4}$, residu 0, quocient 5; $21 \,\underline{|4}$, residu 1, quocient 5; $22 \,\underline{|4}$, residu 2, quocient 5; $23 \,\underline{|4}$, residu 3, quocient 5

tenim $i^n=i^r$ on r és el residu de dividir n entre 4.

(Cicle 4)

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29. a) $(6-5i)+(2-i)-2(-5+6i) = 6-5i+2-i+10-12i = 18-18i$

b) $(2-3i)-(5+4i)+\frac{1}{2}(6-4i) = 2-3i-5-4i+3-2i = -9i$

c) $(3+2i)(4-2i) = 12-6i+8i-4i^2 = 12-6i+8i+4 = 16+2i$

d) $(2+3i)(5-6i) = 10-12i+15i-18i^2 = 10-12i+15i+18 = 28+3i$

e) $(-i+1)(3-2i)(1+3i) = (-3i+2i^2+3-2i)(1+3i) = (-3i-2+3-2i)\cdot(1+3i)$

$= (1-5i)(1+3i) = 1+3i-5i-15i^2 = 1+3i-5i+15 = 16-2i$

f) $\frac{2+4i}{4-2i}\cdot\frac{4+2i}{4+2i} = \frac{(2+4i)(4+2i)}{4^2-2^2\cdot i^2} = \frac{8+4i+16i-8}{16+4} = \frac{20i}{20} = i$

g) $\frac{1-4i}{3+i}\cdot\frac{3-i}{3-i} = \frac{(1-4i)(3-i)}{3^2-i^2} = \frac{3-i-12i-4}{10} = \frac{-1-13i}{10} = \frac{-1}{10}-\frac{13}{10}i$

h) $\frac{4+4i}{-3+5i}\cdot\frac{-3-5i}{-3-5i} = \frac{(4+4i)(-3-5i)}{(-3)^2-5^2\cdot i^2} = \frac{-12-20i-12i+20}{9+25} = \frac{8-32i}{34} =$

$= \frac{4}{17}-\frac{16}{17}i$

i) $\frac{5+i}{-2-i}\cdot\frac{-2+i}{-2+i} = \frac{-10+5i-2i-1}{4+1} = \frac{-11+3i}{5} = -\frac{11}{5}+\frac{3}{5}i$

j) $\frac{1+5i}{3+4i}\cdot\frac{3-4i}{3-4i} = \frac{3-4i+15i+20}{9+16} = \frac{23}{25}+\frac{11}{25}i$

k) $\frac{4-2i}{i}\cdot\frac{-i}{-i} = \frac{4i+2}{-1} = -2-4i$

l) $6-3\left(5+\frac{2}{5}i\right) = 6-15-\frac{6}{5}i = -9-\frac{6}{5}i$

m) $\frac{(-3i)^2(1-2i)}{2+2i} = \frac{-9(1-2i)}{2+2i} = \frac{-(9-18i)}{2+2i}\cdot\frac{2-2i}{2-2i} = \frac{-(18-18i-36i-36)}{4+4} =$

$= +\frac{18}{8}+\frac{54}{8}i = +\frac{9}{4}+\frac{27}{4}i$

30. a) $2+\sqrt{3}i$ i $2-\sqrt{3}i \rightarrow (x-2-\sqrt{3}i)(x-2+\sqrt{3}i) = x^2-4x+7$

$(x-(2+\sqrt{3}i))(x-(2-\sqrt{3}i)) = x^2-2x+\sqrt{3}ix-2x+4-2\sqrt{3}i-\sqrt{3}ix+3+2\sqrt{3}i$

b) $-3i$ i $3i \rightarrow (x+3i)(x-3i) = x^2-(3i)^2 = x^2+9$

c) $1+2i$ i $3-4i \rightarrow (x-(1+2i))(x-(3-4i)) = (x-1-2i)(x-3+4i) =$

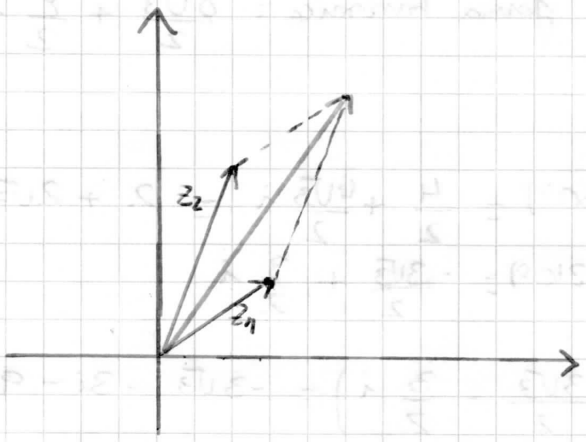
$= x^2-3x+4ix-x+3-4i-2ix+6i+8 =$

$= x^2+(-4+2i)x+(11+2i)$

31. $(25-xi)^2 = 625-50xi-x^2 \xrightarrow{\text{part real}} -x^2+625=0 \rightarrow x = \pm\sqrt{625} \begin{cases} x_1=-25 \\ x_2=+25 \end{cases}$

part imaginară $-50xi$

32. $z_1 = 3+2i$ $\quad z_2 = 2+5i$ $\quad z_1+z_2 = 5+7i$



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33. a) $1+\sqrt{3}i$ $\quad r = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$ $\quad tg\,\alpha = \frac{\sqrt{3}}{1} \to \alpha = 60°$

1r quadrant $\quad = 2_{60°}$

b) $\sqrt{3}+i = 2_{30°}$ $\quad r = \sqrt{\sqrt{3}^2+1^2} = \sqrt{4} = 2$ $\quad tg\,\alpha = \frac{1}{\sqrt{3}} \to \alpha = 30°$

1r quadrant

c) $-1+i = \sqrt{2}_{135°}$ $\quad r = \sqrt{(-1)^2+1^2} = \sqrt{2}$ $\quad tg\,\alpha = \frac{1}{-1} \to \alpha = 45°$ 1r q.

2n quadrant $\quad \alpha = 135°$ 2n q

d) $5-12i = 13_{292°37'}$ $\quad r = \sqrt{5^2+12^2} = 13$ $\quad tg\,\alpha = \frac{-12}{5}$ $\quad \alpha = -67,38°$

4t quadrant $\quad \alpha = 292'62° = 292°37'$

e) $3i = 3_{90°}$ $\quad r = \sqrt{0^2+3^2} = 3$ $\quad \alpha = 90°$

f) $-5 = 5_{180°}$ $\quad r = \sqrt{(-5)^2+0^2} = 5$

34. a) $5_{(\pi/6)\,rad} = 5_{30°} = \frac{5\sqrt{3}}{2}+\frac{5}{2}i$ $\quad a = 5\cdot\cos 30° = \frac{5\sqrt{3}}{2}$ $\quad b = 5\cdot\sin 30° = \frac{5}{2}$

b) $2_{135°} = -\sqrt{2}+\sqrt{2}i$ $\quad a = 2\cdot\cos 135° = -\sqrt{2}$ $\quad b = 2\cdot\sin 135° = \sqrt{2}$

c) $2_{495°} = -\sqrt{2}+\sqrt{2}i$ $\quad a = 2\cdot\cos 495° = -\sqrt{2}$ $\quad b = 2\cdot\sin 495° = \sqrt{2}$

d) $3_{240°} = -\frac{3}{2}-\frac{3\sqrt{3}}{2}i$ $\quad a = 3\cos 240° = -\frac{3}{2}$ $\quad b = 3\sin 240° = -\frac{3\sqrt{3}}{2}$

e) $5_{180°} = -5$ $\quad a = 5\cdot\cos 240° = -5$ $\quad b = 5\cdot\sin 240° = 0$

f) $4_{90°} = 4i$ $\quad a = 4\cdot\cos 90° = 0$ $\quad b = 4\cdot\sin 90° = 1$

35. $z = r_\alpha$ $\quad$ Oposat: $-z = r_{180°+\alpha}$

Conjugat: $\bar{z} = r_{360°-\alpha}$

36. $z = 8(\cos 30^\circ + i \sin 30^\circ)$

forma polară: 8_{30° forma binomică: $\frac{8\sqrt{3}}{2} + \frac{8}{2}i = 4\sqrt{3} + 4i$

37. $z_1 = 4_{60^\circ}$ $z_2 = 3_{210^\circ}$

a) $z_1 = 4(\cos 60^\circ + \sin 60^\circ) = \frac{4}{2} + \frac{4\sqrt{3}}{2}i = 2 + 2\sqrt{3}i$

$z_2 = 3(\cos 210^\circ + \sin 210^\circ) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

b) $z_1 \cdot z_2 = (2 + 2\sqrt{3}i)\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right) = -3\sqrt{3} - 3i - 9i + 3\sqrt{3} = -12i$

$z_1 \cdot z_2 = 12_{270^\circ}$

$\frac{z_2}{z_1} = \frac{-\frac{3\sqrt{3}}{2} - \frac{3}{2}i}{2 + 2\sqrt{3}i} \cdot \frac{2 - 2\sqrt{3}i}{2 - 2\sqrt{3}i} = \frac{-3\sqrt{3} + 9i - 3i - 3\sqrt{3}}{4 + 4 \cdot 3} =$

$= \frac{-6\sqrt{3} + 6i}{16} = \frac{-3\sqrt{3} + 3i}{8}$ (2n quadrant)

$r = \sqrt{\left(-\frac{3\sqrt{3}}{8}\right)^2 + \left(\frac{3}{8}\right)^2} = \frac{3}{4}$ $\quad tg\,\alpha = \frac{\frac{3}{8}}{-\frac{3\sqrt{3}}{8}} = \frac{-1}{\sqrt{3}}$ $\quad \alpha = -30^\circ$

$\alpha = 150^\circ$

$\frac{z_2}{z_1} = \left(\frac{3}{4}\right)_{150^\circ}$

c) $z_1 \cdot z_2 = r_1 r_2{}_{\alpha_1 + \alpha_2}$ $\quad \frac{z_2}{z_1} = \left(\frac{r_2}{r_1}\right)_{\alpha_2 - \alpha_1}$

$z_1 \cdot z_2 = 4_{60^\circ} \cdot 3_{210^\circ} = (4 \cdot 3)_{60^\circ + 210^\circ} = 12_{270^\circ}$ $\quad \frac{z_2}{z_1} = \frac{3_{210^\circ}}{4_{60^\circ}} = \left(\frac{3}{4}\right)_{210^\circ - 60^\circ} = \left(\frac{3}{4}\right)_{150^\circ}$

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38. $0'\widehat{12} = \frac{12 - 0}{99} = \frac{12}{99} = \frac{4}{33}$ $\quad 5'\widehat{6} = \frac{56 - 5}{9} = \frac{51}{9} = \frac{17}{3}$

$0'\widehat{23} = \frac{23 - 2}{90} = \frac{21}{90} = \frac{7}{30}$ $\quad 3'1 = \frac{31}{10}$

$0'12 - 5'\widehat{6} - 0'\widehat{23} + 3{,}1 = \frac{4}{33} - \frac{17}{3} - \frac{7}{30} + \frac{31}{10} = \frac{40}{330} - \frac{1870}{330} - \frac{77}{330} + \frac{1023}{330}$

$= \frac{-884}{330} = \frac{-442}{165} = -2'6\widehat{78}$

39. $4'0\widehat{9} = \frac{409 - 40}{90} = \frac{369}{90} = \frac{123}{30} = \frac{41}{10}$ $\quad 1'3\widehat{9} = \frac{139 - 13}{90} = \frac{126}{90} = \frac{21}{15} = \frac{7}{5}$

$4'0\widehat{9} \cdot 1'3\widehat{9} = \frac{41}{10} \cdot \frac{7}{5} = \frac{287}{50} = 5'74$

40. a) $\sqrt{1,\overline{7}} = \sqrt{\frac{16}{9}} = \frac{4}{3} = 1'\overline{3}$ b) $\sqrt{\frac{1'\overline{3}}{3}} \cdot \sqrt{\frac{4/3}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0'\overline{6}$

$1'\overline{7} = \frac{17-1}{9} = \frac{16}{9}$ $1'\overline{3} = \frac{13-1}{9} = \frac{12}{9} = \frac{4}{3}$

41. a) $\frac{140}{99} < \sqrt{2}$ b) $0'52\overline{6} > 0'\overline{526}$

$\frac{140}{99} = 1'\overline{41}$, $\sqrt{2} = 1'414213...$

c) $4,\overline{89} > 2\sqrt{6}$ d) $-2'098 > -2'1$

$2\sqrt{6} = 4'898979...$

42. $F = \sqrt{\sqrt{2}^2 + 1^2} = \sqrt{3}$ $H = \sqrt{\sqrt{5}^2 + 1^2} = \sqrt{6}$

↑

$\sqrt{2^2 + 1^2} = \sqrt{5}$

43. $a = \frac{2}{7}$ $b = \frac{4}{7}$ $c = \frac{5}{7}$ $d = \frac{-1}{7}$

44. $\left(\frac{3}{2} - \frac{3}{4}\right)^{-2}\left(\frac{1}{3} - \frac{7}{9}\right)^{-1} + 4 = \left(\frac{6}{4} - \frac{3}{4}\right)^{-2}\left(\frac{3}{9} - \frac{7}{9}\right)^{-1} + 4 =$

$= \left(\frac{3}{4}\right)^{-2} \cdot \left(-\frac{4}{9}\right)^{-1} + 4 = \left(\frac{4}{3}\right)^2 \cdot \left(-\frac{9}{4}\right) + 4 = -\frac{16}{9} \cdot \frac{9}{4} + 4 = -4 + 4 = 0$

45. a) $\frac{3^6 \cdot 2^5 \cdot 5^2}{9^3 \cdot 4^3 \cdot 5} = \frac{3^6 \cdot 2^5 \cdot 5^2}{(3^2)^3 (2^2)^3 \cdot 5} = \frac{3^6 \cdot 2^5 \cdot 5^2}{3^6 \cdot 2^6 \cdot 5} = \frac{5}{2}$

b) $\frac{3^4 \cdot 16 \cdot 9^{-1}}{5^{-1} \cdot 3^5} = \frac{3^4 \cdot 2^4 \cdot (3^2)^{-1}}{5^{-1} \cdot 3^5} = \frac{3^4 \cdot 2^4 \cdot 5}{3^2 \cdot 3^5} = \frac{3^4 \cdot 2^4 \cdot 5}{3^7} = \frac{2^4 \cdot 5}{3^3} = \frac{80}{27}$

c) $\frac{15^2 \cdot 8^{-1}}{6^3 \cdot 10^2} = \frac{5^2 \cdot 3^2 \cdot 2^{-3}}{2^3 \cdot 3^3 \cdot 2^2 \cdot 5^2} = \frac{5^2 \cdot 3^2}{2^3 \cdot 2^3 \cdot 3^3 \cdot 2^2 \cdot 5^2} = \frac{5^2 \cdot 3^2}{2^8 \cdot 3^3 \cdot 5^2} = \frac{1}{2^8 \cdot 3} = \frac{1}{768}$

d) $\frac{a^{-3} b^{-4} c^7}{a^{-5} b^2 c^{-1}} = \frac{a^5 c^7 c}{a^3 b^2 b^4} = \frac{a^2 c^8}{b^6}$

46. a) $\sqrt[5]{a^2} \cdot \sqrt{a} = a^{\frac{2}{5}} a^{\frac{1}{2}} = a^{\frac{2}{5} + \frac{1}{2}} = a^{\frac{4}{10} + \frac{5}{10}} = a^{\frac{9}{10}} = \sqrt[10]{a^9}$

b) $\frac{\sqrt[3]{x^2}}{\sqrt{x}} = \frac{x^{\frac{2}{3}}}{x^{\frac{1}{2}}} = x^{\frac{2}{3} - \frac{1}{2}} = x^{\frac{4}{6} - \frac{3}{6}} = x^{\frac{1}{6}} = \sqrt[6]{x}$

c) $\frac{1}{\sqrt[4]{a^3}} = \frac{1}{a^{3/4}} = a^{-3/4} = \sqrt[4]{a^{-3}}$

47. a) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$ b) $\sqrt[3]{343} = \sqrt[3]{7^3} = 7$ c) $\sqrt[4]{625} = \sqrt[4]{5^4} = 5$

d) $\sqrt{0'25} = \sqrt{\frac{1}{4}} = \sqrt{\left(\frac{1}{2}\right)^2} = \frac{1}{2} = 0,5$ e) $\sqrt[3]{8^4} = \sqrt[3]{(2^3)^4} = 2^4 = 16$ f) $\sqrt[3]{0'001} = \sqrt[3]{10^{-3}} = 10^{-1} = 0'1$

48. a) $\frac{1}{\sqrt{2}} = 2^{-\frac{1}{2}}$ b) $(-32)^{1/5} = [(-2)^5]^{\frac{1}{5}} = (-2)^1 = -2$ c) $(\sqrt[8]{2})^4 = 2^{\frac{4}{8}} = 2^{\frac{1}{2}}$

49. a) $4 \cdot \frac{1}{3} \cdot \left(-\frac{3}{2}\right)^3 = -\frac{2^2 \cdot 3^3}{3 \cdot 2^3} = -\frac{3^2}{2} = -\frac{9}{2}$

b) $\left(-\frac{1}{2}\right)^4 \cdot \left(\frac{2}{9}\right)^{-1} \cdot \frac{1}{8} = \frac{(-1)^4 \cdot 2^{-1} \cdot 1}{2^4 \cdot 9^{-1} \cdot 8} = \frac{1}{2 \cdot 2^4 \cdot 3^2 \cdot 2^3} = \frac{3^2}{2^8} = \frac{9}{256}$

c) $\frac{(-5)^3 (-8)^3 (-9)^2}{15^2 \cdot 20^4} = \frac{5^3 \cdot 2^9 \cdot 3^4}{5^2 \cdot 3^2 \cdot 5^4 \cdot 2^8} = \frac{2 \cdot 3^2}{5^3} = \frac{18}{125}$

d) $\frac{(-30)^{-1} \cdot 15^2}{10^3} = -\frac{5^{-1} \cdot 3^{-1} \cdot 2^{-1} \cdot 5^2 \cdot 3^2}{2^3 \cdot 5^3} = \frac{-5^2 \cdot 3^2}{5 \cdot 3 \cdot 2 \cdot 2^3 \cdot 5^3} = \frac{-5^2 \cdot 3^2}{2^4 \cdot 3 \cdot 5^4} = \frac{-3}{2^4 \cdot 5^2} = \frac{-3}{400}$

50. a) $\frac{\sqrt[4]{a^3} \cdot a^{-1}}{a\sqrt{a}} = \frac{a^{\frac{3}{4}}}{a \cdot a \cdot a^{1/2}} = \frac{a^{3/4}}{a^{5/2}} = \frac{a^{3/4}}{a^{10/4}} = a^{-\frac{7}{4}} = \frac{1}{\sqrt[4]{a^7}} = \frac{1}{a\sqrt[4]{a^3}}$

b) $16^{1/4} \cdot \sqrt[3]{\frac{1}{4}} \cdot \frac{1}{\sqrt[6]{4}} = 2^{\frac{7}{4}} \cdot \frac{1}{2^{2/3}} \cdot \frac{1}{2^{2/6}} = \frac{2}{2^{2/3} \cdot 2^{1/3}} = \frac{2}{2} = 1$

51. a) $\frac{a^2 \cdot b^{-2}}{a^{-2} \cdot b^2} = 1$ Falsch.

$\frac{a^2 \cdot a^2}{b^2 \cdot b^2} = \frac{a^4}{b^4}$

b) $(3^{-2})^{-3} \left(\frac{1}{27}\right)^2 = 1$

$3^6 \cdot \left(\frac{1}{3^3}\right)^2 = 1$

$3^6 \cdot \frac{1}{3^6} = 1$

$\frac{3^6}{3^6} = 1$ Gut

c) $\frac{3^2 - 5^{-2}}{3^{-1} - 5^{-1}} = \frac{8}{15}$

$\frac{\frac{1}{3^2} - \frac{1}{5^2}}{\frac{1}{3} - \frac{1}{5}} = \frac{8}{15}$

$\frac{\frac{1}{9} - \frac{1}{25}}{\frac{1}{3} - \frac{1}{5}} = \frac{8}{15}$

$\frac{\frac{25-9}{225}}{\frac{53}{15}} = \frac{8}{15}$

$\frac{\frac{16}{225}}{\frac{2}{15}} = \frac{8}{15}$

$\frac{8}{15} = \frac{8}{15}$ Gut

d) $\left(\frac{1}{3}\right)^{-2} - (-3)^{-2} = \frac{80}{9}$

$3^2 - \frac{1}{(-3)^2} = \frac{80}{9}$

$9 - \frac{1}{9} = \frac{80}{9}$

$\frac{81}{9} - \frac{1}{9} = \frac{80}{9}$

$\frac{80}{9} = \frac{80}{9}$ Gut

52. a) $(0{,}125)^{1/3} = 2^{-1}$

$\left(\frac{1}{8}\right)^{1/3} = 2^{-1}$

$\left(\frac{1}{2^3}\right)^{1/3} = 2^{-1}$

$\frac{1}{2} = 2^{-1}$

b) $(0{,}25)^{-1/2} = 2$

$\left(\frac{1}{4}\right)^{-1/2} = 2$

$\left(\frac{1}{2^2}\right)^{-1/2} = 2$

$(2^2)^{1/2} = 2$

$2 = 2$

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53. a) $2\sqrt[3]{3} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{24}$ b) $4\sqrt[3]{\frac{1}{4}} = \sqrt[3]{\frac{4^3}{4}} = \sqrt[3]{4^2} = \sqrt[3]{16}$

c) $\frac{2}{x}\sqrt{\frac{3x}{8}} = \sqrt{\frac{2^2 \cdot 3x}{x^2 \cdot 8}} = \sqrt{\frac{3}{2x}}$ d) $\frac{3}{5}\sqrt[3]{\frac{25}{9}} = \sqrt[3]{\frac{3^3 \cdot 25}{5^3 \cdot 9}} = \sqrt[3]{\frac{3}{5}}$

e) $2\sqrt[4]{4} = \sqrt[4]{2^4 \cdot 4} = \sqrt[4]{2^6} = \sqrt{2^3} = \sqrt{8}$ f) $\frac{1}{5}\sqrt[3]{15} = \sqrt[3]{\frac{15}{5^3}} = \sqrt[3]{\frac{3}{25}}$

54. a) $\sqrt[3]{16} = \sqrt[3]{2^4} = 2\sqrt[3]{2}$ b) $4\sqrt{8} = 4\sqrt{2^3} = 4 \cdot 2\sqrt{2} = 8\sqrt{2}$

c) $\sqrt{1000} = \sqrt{10^3} = 10\sqrt{10}$ d) $\sqrt[3]{8a^5} = \sqrt[3]{2^3 \cdot a^5} = 2 \cdot a\sqrt[3]{a^2}$

e) $\sqrt{\frac{125a^2}{16b}} = \sqrt{\frac{5^3 a^2}{2^4 b}} = \frac{5a}{2^2}\sqrt{\frac{5}{b}} = \frac{5a}{4}\sqrt{\frac{5}{b}}$ f) $\sqrt{\frac{1}{4} + \frac{1}{9}} = \sqrt{\frac{9}{36} + \frac{4}{36}} = \sqrt{\frac{13}{36}} = \frac{1}{6}\sqrt{13}$

g) $\sqrt{\frac{16}{a^3}} = \frac{4}{a}\sqrt{\frac{1}{a}}$ h) $\sqrt{4a^2+4} = \sqrt{4(a^2+1)} = 2\sqrt{a^2+1}$

i) $\sqrt{\frac{a}{9} + \frac{a}{16}} = \sqrt{\frac{16a+9a}{144}} = \sqrt{\frac{25a}{144}} = \frac{5}{12}\sqrt{a}$

55. a) $\sqrt[6]{0{,}027} = \sqrt[6]{3^3 \cdot 10^{-3}} = \sqrt[6]{\left(\frac{3}{10}\right)^3} = \sqrt{\frac{3}{10}} = \sqrt{3 \cdot 10^{-1}} = \sqrt{0{,}3}$ $\left(= \sqrt{\frac{3}{10}}\right)$

b) $\sqrt[8]{0{,}0016} = \sqrt[8]{2^4 \cdot 10^{-4}} = \sqrt[8]{\frac{2^4}{10^4}} = \sqrt[8]{\left(\frac{2}{10}\right)^4} = \sqrt{\frac{2}{10}} = \sqrt{2 \cdot 10^{-1}} = \sqrt{0{,}2}$ $\left(= \sqrt{\frac{1}{5}}\right)$

c) $\sqrt[4]{1 + \frac{9}{16}} = \sqrt[4]{\frac{16}{16} + \frac{9}{16}} = \sqrt[4]{\frac{25}{16}} = \sqrt[4]{\frac{5^2}{4^2}} = \sqrt[4]{\left(\frac{5}{4}\right)^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

56. a) $\sqrt[3]{24} = \sqrt[3]{2^3 \cdot 3} = 2\sqrt[3]{3}$ b) $\sqrt[6]{27} = \sqrt[6]{3^3} = \sqrt{3}$

c) $\sqrt[3]{-108} = -\sqrt[3]{2^2 \cdot 3^3} = -3\sqrt[3]{4}$ d) $\sqrt[12]{64y^3} = \sqrt[12]{2^6 y^3} = \sqrt[4]{2^2 y} = \sqrt[4]{4y} = \sqrt{2}\sqrt[4]{y}$

e) $\sqrt[8]{625} : \sqrt[4]{25} = \sqrt[8]{5^4} : \sqrt[4]{5^2} = \sqrt{5} : \sqrt{5} = 1$

57. a) $\sqrt[4]{4}, \sqrt[3]{3}, \sqrt{2}$

$\sqrt[12]{4^3}\ \sqrt[12]{3^4}\ \sqrt[12]{2^6}$

$\sqrt[12]{64}\ \sqrt[12]{81}\ \sqrt[12]{64}$

$\sqrt[4]{4} = \sqrt{2} < \sqrt[3]{3}$

b) $\sqrt{6}, \sqrt[3]{4}$

$\sqrt[6]{6^3}, \sqrt[6]{4^2}$

$\sqrt[6]{216}, \sqrt[6]{16}$

$\sqrt[3]{4} < \sqrt{6}$

c) $\sqrt[4]{6}, \sqrt[5]{10}$

$\sqrt[20]{6^5}\ \sqrt[20]{10^4}$

$\sqrt[20]{7776}\ \sqrt[20]{10000}$

$\sqrt[4]{6} < \sqrt[5]{10}$

d) $\sqrt[4]{72}, \sqrt[3]{9}, \sqrt[6]{100}$

$\sqrt[12]{72^3}\ \sqrt[12]{9^4}\ \sqrt[12]{100^2}$

$\sqrt[12]{373248}\ \sqrt[12]{6561}\ \sqrt[12]{10000}$

$\sqrt[3]{9} < \sqrt[6]{100} < \sqrt[4]{72}$

58. a) $4\sqrt{27} \cdot 5\sqrt{6} = 20\sqrt{3^3} \cdot \sqrt{2 \cdot 3} = 20\sqrt{3^4 \cdot 2} = 20 \cdot 3^2\sqrt{2} = 180\sqrt{2}$

b) $2\sqrt{\frac{4}{3}} \cdot \sqrt{\frac{27}{8}} = 2 \cdot \sqrt{\frac{2^2}{3}} \cdot \sqrt{\frac{3^3}{2^3}} = 2 \cdot \sqrt{\frac{2^2 \cdot 3^3}{3 \cdot 2^3}} = 2 \cdot \sqrt{\frac{3^2}{2}} = 2 \cdot 3\sqrt{\frac{1}{2}} = 6\sqrt{\frac{1}{2}}$

c) $\sqrt{2} \cdot \sqrt{\frac{1}{8}} = \sqrt{\frac{2}{2^3}} = \sqrt{\frac{1}{2^2}} = \frac{1}{2}$

d) $(\sqrt[3]{12})^2 = \sqrt[3]{(2^2\cdot 3)^2} = \sqrt[3]{2^4\cdot 3^2} = 2\sqrt[3]{2\cdot 3^2} = 2\sqrt[3]{18}$

e) $(\sqrt[6]{32})^3 = \sqrt[6]{(2^5)^3} = \sqrt[6]{2^{15}} = 2^2\sqrt[6]{2^3} = 4\sqrt{2}$

f) $\sqrt[3]{24} : \sqrt[3]{3} = \sqrt[3]{2^3\cdot 3} : \sqrt[3]{3} = \sqrt[3]{\frac{2^3\cdot 3}{3}} = \sqrt[3]{2^3} = 2$

59. a) $\sqrt[3]{2}\cdot\sqrt{3} = \sqrt[6]{2^2}\cdot\sqrt[6]{3^3} = \sqrt[6]{2^2\cdot 3^3} = \sqrt[6]{108}$

b) $\sqrt[3]{a}\cdot\sqrt[3]{\frac{1}{a}}\cdot\sqrt{a} = \sqrt[6]{a^2}\cdot\sqrt[6]{\frac{1}{a^2}}\cdot\sqrt[6]{a^3} = \sqrt[6]{\frac{a^2\cdot 1\cdot a^3}{a^2}} = \sqrt[6]{a^3} = \sqrt{a}$

↳ $\sqrt[3]{\frac{a}{a}}\cdot\sqrt{a} = \sqrt[3]{1}\cdot\sqrt{a} = \sqrt{a}$

c) $\left(\frac{\sqrt[6]{32}}{\sqrt{8}}\right)^3 = \left(\frac{\sqrt[6]{2^5}}{\sqrt{2^3}}\right)^3 = \left(\sqrt[6]{\frac{2^5}{2^9}}\right)^3 = \left(\sqrt[6]{\frac{1}{2^4}}\right)^3 = \sqrt[6]{\frac{1}{2^{12}}} = \frac{1}{2^2} = \frac{1}{4}$

d) $\sqrt[3]{2\sqrt{3}} : \sqrt{\sqrt[3]{4}} = \sqrt[3]{\sqrt{2^2\cdot 3}} : \sqrt[6]{4} = \sqrt[6]{2^2\cdot 3} : \sqrt[6]{2^2} = \sqrt[6]{\frac{2^2\cdot 3}{2^2}} = \sqrt[6]{3}$

60. a) $\sqrt[4]{\sqrt[3]{4}} = \sqrt[12]{2^2} = \sqrt[6]{2}$ b) $\sqrt[3]{2\sqrt[4]{8}} = \sqrt[3]{\sqrt[4]{2^4\cdot 2^3}} = \sqrt[12]{2^7}$

c) $(\sqrt[4]{a^3}\cdot\sqrt[5]{a^4}) : \sqrt{a} = \sqrt[20]{\frac{a^{15}\cdot a^{16}}{a^{10}}} = \sqrt[20]{\frac{a^{31}}{a^{10}}} = \sqrt[20]{a^{21}} = a\sqrt[20]{a}$

61. a) $\frac{2\sqrt{3}}{\sqrt{18}} = \frac{2\sqrt{3}}{\sqrt{2\cdot 3^2}} = \frac{2\sqrt{3}}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{6}}{3\cdot 2} = \frac{\sqrt{6}}{3}$

b) $\frac{2}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$

c) $\frac{\sqrt{2}-1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{(\sqrt{2}-1)(\sqrt{2})}{2} = \frac{2-\sqrt{2}}{2}$

d) $\frac{3}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{3(3-\sqrt{3})}{9-3} = \frac{3(3-\sqrt{3})}{6} = \frac{3-\sqrt{3}}{2}$

e) $\frac{\sqrt{72}+3\sqrt{32}-\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{72}+3\sqrt{32}-\sqrt{8}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2^3\cdot 3^2\cdot 2}+3\cdot\sqrt{2^5\cdot 2}-\sqrt{2^3\cdot 2}}{2\cdot 2} =$

$= \frac{\sqrt{2^4\cdot 3^2}+3\cdot\sqrt{2^6}-\sqrt{2^4}}{4} = \frac{2^2\cdot 3+3\cdot 2^3-2^2}{4} = \frac{12+24-4}{4} = \frac{32}{4} = 8$

62. a) $5\sqrt{125}+6\sqrt{45}-7\sqrt{20}+\frac{3}{2}\sqrt{80} = 5\cdot\sqrt{5^3}+6\sqrt{3^2\cdot 5}-7\sqrt{2^2\cdot 5}+\frac{3}{2}\sqrt{2^4\cdot 5} =$

$= 5\,5\sqrt{5}+6\,3\sqrt{5}-7\cdot 2\sqrt{5}+\frac{3}{2}\cdot 2^2\sqrt{5} = 25\sqrt{5}+18\sqrt{5}-14\sqrt{5}+6\sqrt{5} =$

$= 35\sqrt{5}$

b) $\sqrt[3]{16}+2\sqrt[3]{2}-\sqrt[3]{54}-\frac{21}{5}\sqrt[3]{250} = \sqrt[3]{2^4}+2\sqrt[3]{2}-\sqrt[3]{3^3\cdot 2}-\frac{21}{5}\sqrt[3]{2\cdot 5^3} =$

$= 2\sqrt[3]{2}+2\sqrt[3]{2}-3\sqrt[3]{2}-\frac{21}{5}\cdot 5\sqrt[3]{2} = 2\sqrt[3]{2}+2\sqrt[3]{2}-3\sqrt[3]{2}-21\sqrt[3]{2} = -20\sqrt[3]{2}$

c) $\sqrt{125}+\sqrt{54}-\sqrt{45}-\sqrt{24}=\sqrt{5^3}+\sqrt{2\cdot 3^3}-\sqrt{3^2\cdot 5}-\sqrt{2^3\cdot 3}=$

$=5\sqrt{5}+3\sqrt{2\cdot 3}-3\sqrt{5}-2\sqrt{2\cdot 3}=2\sqrt{5}+\sqrt{6}$

d) $(\sqrt{2}+\sqrt{3})(\sqrt{6}-1)=\sqrt{12}-\sqrt{2}+\sqrt{18}-\sqrt{3}=\sqrt{2^2\cdot 3}-\sqrt{2}+\sqrt{2\cdot 3^2}-\sqrt{3}=$

$=2\sqrt{3}-\sqrt{2}+3\sqrt{2}-\sqrt{3}=\sqrt{3}+2\sqrt{2}$

63. a) $3\sqrt[3]{16}-2\sqrt[3]{250}+5\sqrt[3]{54}-4\sqrt[3]{2}=3\sqrt[3]{2^4}-2\sqrt[3]{5^3\cdot 2}+5\sqrt[3]{3^3\cdot 2}-4\sqrt[3]{2}=$

$=3\cdot 2\sqrt[3]{2}-2\cdot 5\sqrt[3]{2}+5\cdot 3\sqrt[3]{2}-4\sqrt[3]{2}=6\sqrt[3]{2}-10\sqrt[3]{2}+15\sqrt[3]{2}-4\sqrt[3]{2}=$

$=7\sqrt[3]{2}$

b) $\sqrt{\frac{2}{5}}-4\cdot\sqrt{\frac{18}{125}}+\frac{1}{3}\sqrt{\frac{8}{45}}=\sqrt{\frac{2}{5}}-4\cdot\sqrt{\frac{3^2\cdot 2}{5^3}}+\frac{1}{3}\sqrt{\frac{2^3}{3^2\cdot 5}}=$

$=\sqrt{\frac{2}{5}}-\frac{4\cdot 3}{5}\sqrt{\frac{2}{5}}+\frac{1}{3}\cdot\frac{2}{3}\sqrt{\frac{2}{5}}=\sqrt{\frac{2}{5}}-\frac{12}{5}\sqrt{\frac{2}{5}}+\frac{2}{9}\sqrt{\frac{2}{5}}=$

$=\left(1-\frac{12}{5}+\frac{2}{9}\right)\sqrt{\frac{2}{5}}=\left(\frac{45-108+10}{45}\right)\sqrt{\frac{2}{5}}=\frac{-53}{45}\sqrt{\frac{2}{5}}$

c) $7\sqrt[3]{81a}-2\sqrt[3]{3a^4}+\frac{\sqrt[3]{3a}}{5}=7\sqrt[3]{3^4a}-2\sqrt[3]{3a^4}+\frac{1}{5}\sqrt[3]{3a}=$

$=7\cdot 3\sqrt[3]{3a}-2a\sqrt[3]{3a}+\frac{1}{5}\sqrt[3]{3a}=21\sqrt[3]{3a}-2a\sqrt[3]{3a}+\frac{1}{5}\sqrt[3]{3a}=$

$=\left(21-2a+\frac{1}{5}\right)\sqrt[3]{3a}=\left(\frac{105+1}{5}-2a\right)\sqrt[3]{3a}=\left(\frac{106}{5}-2a\right)\sqrt[3]{3a}$

64. a) $(\sqrt{3}+\sqrt{2})^2-(\sqrt{3}-\sqrt{2})^2=(3+2+2\sqrt{6})-(3+2-2\sqrt{6})=$

$=3+2+2\sqrt{6}-3-2+2\sqrt{6}=4\sqrt{6}$

b) $(\sqrt{6}+\sqrt{5})\cdot 2\cdot\sqrt{2}=2(\sqrt{2\cdot 3\cdot 2}+\sqrt{5\cdot 2})=2\cdot(2\sqrt{3}+\sqrt{10})=$

$=4\sqrt{3}+2\sqrt{10}$

c) $(\sqrt{5}-\sqrt{6})(\sqrt{5}+\sqrt{6})=5-6=-1$

d) $(2\sqrt{5}-3\sqrt{2})^2=4\cdot 5+9\cdot 2-12\sqrt{5\cdot 2}=20+18-12\sqrt{10}=38-12\sqrt{10}$

e) $(\sqrt{2}-1)(\sqrt{2}+1)\sqrt{3}=(2-1)\sqrt{3}=\sqrt{3}$

65. a) $\frac{2\sqrt{3}-\sqrt{2}}{\sqrt{18}}=\frac{2\sqrt{3}-\sqrt{2}}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{6}-2}{3\cdot 2}=\frac{2(\sqrt{6}-1)}{6}=\frac{\sqrt{6}-1}{3}$

b) $\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{12}}=\frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\cdot 3+\sqrt{6}}{2\cdot 3}=\frac{6+\sqrt{6}}{6}=1+\frac{\sqrt{6}}{6}$

c) $\frac{1}{2(\sqrt{3}-\sqrt{5})}\cdot\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\frac{\sqrt{3}+\sqrt{5}}{2(3-5)}=\frac{\sqrt{3}+\sqrt{5}}{-4}=\frac{-\sqrt{3}-\sqrt{5}}{4}$

d) $\frac{3}{\sqrt{5}-2}\cdot\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{3(\sqrt{5}+2)}{5-4}=3\sqrt{5}+6$

e) $\frac{11}{-2\sqrt{5}+3} \cdot \frac{3+2\sqrt{5}}{3+2\sqrt{5}} = \frac{11(3+2\sqrt{5})}{9-4\cdot 5} = \frac{11(3+2\sqrt{5})}{11} = 3+2\sqrt{5}$

f) $\frac{3\sqrt{6}+2\sqrt{2}}{3\sqrt{3}+2} \cdot \frac{3\sqrt{3}-2}{3\sqrt{3}-2} = \frac{(3\sqrt{6}+2\sqrt{2})(3\sqrt{3}-2)}{9\cdot 3-4} = \frac{9\sqrt{18}-6\sqrt{6}+6\sqrt{6}-4\sqrt{2}}{23} =$

$= \frac{27\sqrt{2}-6\sqrt{6}+6\sqrt{6}-4\sqrt{2}}{23} = \frac{23\sqrt{2}}{23} = \sqrt{2}$

66. a) $\frac{3}{\sqrt{3}-\sqrt{2}} - \frac{2}{\sqrt{3}+\sqrt{2}} = 3\sqrt{3}+3\sqrt{2}-(2\sqrt{3}-2\sqrt{2}) = \sqrt{3}+5\sqrt{2}$

$\frac{3}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3(\sqrt{3}+\sqrt{2})}{3-2} = 3\sqrt{3}+3\sqrt{2}$

$\frac{2}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2})}{3-2} = 2\sqrt{3}-2\sqrt{2}$

b) $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} - \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = (6-\sqrt{35})-(6+\sqrt{35}) = -2\sqrt{35}$

$\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \cdot \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{7+5-2\sqrt{35}}{7-5} = \frac{12-2\sqrt{35}}{2} = 6-\sqrt{35}$

$\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} \cdot \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{7+5+2\sqrt{35}}{7-5} = \frac{12+2\sqrt{35}}{2} = 6+\sqrt{35}$

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67. $\frac{1}{1-\frac{\sqrt{3}}{1+\sqrt{3}}} + \frac{1}{1+\frac{\sqrt{3}}{1-\sqrt{3}}} = \frac{1}{1-\frac{3-\sqrt{3}}{2}} + \frac{1}{1+\frac{-3-\sqrt{3}}{2}} = \frac{1}{\frac{-1+\sqrt{3}}{2}} + \frac{1}{\frac{-1-\sqrt{3}}{2}} =$

$\frac{\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{\sqrt{3}(1-\sqrt{3})}{1-3} = \frac{\sqrt{3}-3}{-2} = \frac{3-\sqrt{3}}{2}$ | $= \frac{2}{\sqrt{3}-1} - \frac{2}{1+\sqrt{3}} =$

$\frac{\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{\sqrt{3}(1+\sqrt{3})}{1-3} = \frac{\sqrt{3}+3}{-2} = \frac{-3-\sqrt{3}}{2}$ | $= \sqrt{3}+1-(\sqrt{3}-1) = 2$

$\frac{2}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2\sqrt{3}+2}{3-1} = \frac{2(\sqrt{3}+1)}{2} = \sqrt{3}+1$

$\frac{2}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{2-2\sqrt{3}}{1-3} = \sqrt{3}-1$

68. a) $\frac{(3'12\cdot 10^{-5}+7'03\cdot 10^{-4})\cdot 8'3\cdot 10^{8}}{4,32\cdot 10^{3}} = \frac{(0'312\cdot 10^{-4}+7'03\cdot 10^{-4})\cdot 8'3\cdot 10^{8}}{4,32\cdot 10^{3}} =$

$= \frac{(7'342\cdot 10^{-4})\cdot(8'3\cdot 10^{8})}{4'32\cdot 10^{3}} = \frac{7'342\cdot 8'3}{4'32} \cdot \frac{10^{-4}\cdot 10^{8}}{10^{3}} = 14'106\cdot 10 =$

$= 1'4106\cdot 10^{2}$

b) $\frac{(12'5\cdot 10^{7}-8\cdot 10^{9})(75\cdot 10^{-5}+185)}{9,2\cdot 10^{6}} = \frac{(12'5\cdot 10^{7}-800\cdot 10^{7})\cdot(0'000035+185)}{9,2\cdot 10^{6}} =$

$\frac{(-787'5\cdot 10^{7})(185'000035)}{9'2\cdot 10^{6}} = \frac{(-787'5\cdot 185'000035)}{9'2} \cdot \frac{10^{7}}{10^{6}} = -15835'601\cdot 10 =$

$= -1'5835601\cdot 10^{5}$

c) $\frac{5'431\cdot 10^3 - 6'51\cdot 10^4 + 385\cdot 10^2}{8{,}2\cdot 10^{-3} - 2\cdot 10^{-4}} = \frac{5'431\cdot 10^3 - 65'1\cdot 10^3 + 38'5\cdot 10^3}{8{,}2\cdot 10^{-3} - 02\cdot 10^{-3}} =$

$= \frac{-21'169\cdot 10^3}{8\cdot 10^{-3}} = -2'646\ 10^6$

69. a) $3{,}27\cdot 10^{13}$ $\quad 85{,}7\cdot 10^{12} = 8{,}57\cdot 10^{13}$ $\quad 453\cdot 10^{11} = 4{,}53\cdot 10^{13}$

$85{,}7\cdot 10^{12} > 453\cdot 10^{11} > 3{,}27\cdot 10^{13}$

b) $1{,}19\cdot 10^{-9}$ $\quad 0{,}05\cdot 10^{-7} = 5\cdot 10^{-9}$ $\quad 2000\cdot 10^{-12} = 2\cdot 10^{-9}$

$0{,}05\cdot 10^{7} > 2000\cdot 10^{-12} > 1{,}19\cdot 10^{-9}$

70. $\frac{2\cdot 10^{-7} - 3\cdot 10^{-5}}{4\cdot 10^6 + 10^5} = \frac{0'02\cdot 10^{-5} - 3\cdot 10^{-5}}{40\cdot 10^5 + 10^5} = \frac{-2'98\cdot 10^{-5}}{41\cdot 10^5} = -0'07268\cdot 10^{-10} =$

$= -7'268\cdot 10^{-12}$

71. $\frac{60000^3\cdot 0'000024}{100^2\cdot 72000000\cdot 0{,}0002^5} = \frac{6^3\cdot 10^{12}\cdot 2^4\cdot 10^{-20}}{10^4\cdot 7{,}2\cdot 10^7\cdot 2^5\cdot 10^{-20}} =$

$= \frac{2'16\cdot 10^{14}\cdot 1'6\cdot 10^{-19}}{10^4\cdot 7{,}2\cdot 10^7\cdot 3{,}2\cdot 10^{-19}} = 0'15\cdot 10^3 = 1'5\cdot 10^2 = 150$

72. $A = 3'2\cdot 10^7$ $\quad B = 5'28\cdot 10^4$ $\quad C = 2'01\cdot 10^5$

$\frac{B+C}{A} = \frac{5'28\cdot 10^4 + 2'01\cdot 10^5}{3{,}2\cdot 10^7} = \frac{0'528\cdot 10^5 + 2'01\cdot 10^5}{3{,}2\cdot 10^7} = \frac{2'538\cdot 10^5}{3{,}2\cdot 10^7} =$

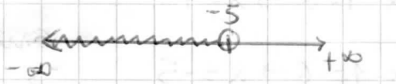
$= 0'793125\cdot 10^{-2} = 7'93125\cdot 10^{-3}$

73. $A = 3'24\cdot 10^6$ $\quad B = 5'1\cdot 10^{-5}$ $\quad C = 3'8\cdot 10^{11}$ $\quad D = 6{,}2\cdot 10^{-6}$

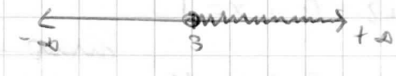
$\left(\frac{A}{B} + C\right)\cdot D = \left(\frac{3'24\cdot 10^6}{5'1\cdot 10^{-5}} + 3'8\cdot 10^{11}\right)\cdot 6{,}2\cdot 10^{-6} = (0'635\cdot 10^{11} + 3'8\cdot 10^{11})\cdot$

$\cdot 6'2\cdot 10^{-6} = 4'435\cdot 10^{11}\cdot 6{,}2\cdot 10^{-6} = 27'499\cdot 10^5 = 2'7499\cdot 10^6$

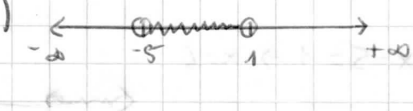
74. a) $\{x \mid x < -5\} = (-\infty, -5)$



b) $\{x \mid 3 \leq x\} = [3, +\infty)$

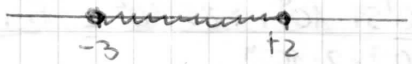


c) $\{x \mid -5 < x < 1\} = (-5, 1)$

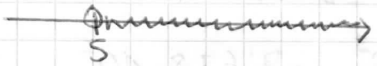


d) $\{x \mid -2 \leq x \leq 0\} = [-2, 0]$

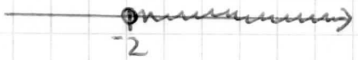
75. a) $-3 \leq x \leq 2 \rightarrow [-3, +2]$



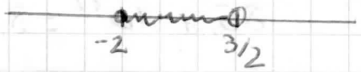
b) $5 < x \rightarrow (5, +\infty)$



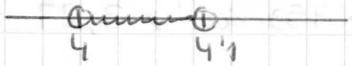
c) $x \geq -2$ $[-2, +\infty)$



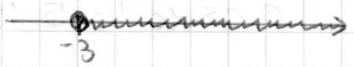
d) $-2 \leq x < \frac{3}{2}$ $[-2, \frac{3}{2})$



e) $4 < x < 4{,}1$ $(4, 4'1)$



f) $-3 \leq x$ $[3, +\infty)$



76. a) $[-2,7] = \{x \mid -2 \leq x \leq 7\}$ b) $[13, +\infty) = \{x \mid 13 \leq x\}$ c) $(-\infty, 0) = \{x \mid x < 0\}$

d) $(-3, 0] = \{x \mid -3 < x \leq 0\}$ e) $[3/2, 6) = \{x \mid \frac{3}{2} \leq x < 6\}$ f) $(-\infty, +\infty) = \mathbb{R}$

77. a) $A = [-3, 2]$ $B = [0, 5]$ $A \cap B = [0, 2]$

b) $I = [2, +\infty)$ $J = (0, 10)$ $I \cap J = [2, 10)$

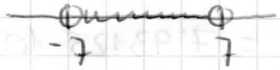
78. a) $x < 3$ o $x \geq 5 \rightarrow (-\infty, 3) \cup [5, +\infty)$

b) $x > 0$ i $x < 4 \rightarrow (0, 4)$

c) $x \leq -1$ o $x > 1 \rightarrow (-\infty, -1] \cup (1, +\infty)$

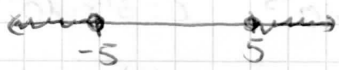
d) $x < 3$ i $x \leq -2 \rightarrow (-\infty, -2]$

79. a) $|x| < 7 \begin{cases} x = 7 \\ x = -7 \end{cases}$



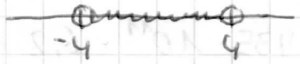
Sol: $(-7, 7)$

b) $|x| \geq 5 \begin{cases} x = 5 \\ x = -5 \end{cases}$



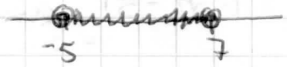
Sol: $(-\infty, -5] \cup [5, +\infty)$

c) $|2x| < 8 \begin{cases} 2x = 8 \rightarrow x = 4 \\ 2x = -8 \rightarrow x = -4 \end{cases}$



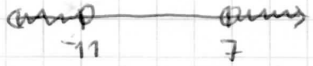
Sol: $(-4, 4)$

d) $|x - 1| \leq 6 \begin{cases} x - 1 = 6 \rightarrow x = 7 \\ x - 1 = -6 \rightarrow x = -5 \end{cases}$



Sol: $[-5, 7]$

e) $|x + 2| > 9 \begin{cases} x + 2 = 9 \rightarrow x = 7 \\ x + 2 = -9 \rightarrow x = -11 \end{cases}$



Sol: $(-\infty, -11) \cup (7, +\infty)$

f) $|x - 5| \geq 1 \begin{cases} x - 5 = 1 \rightarrow x = 6 \\ x - 5 = -1 \rightarrow x = 4 \end{cases}$

Sol: $(-\infty, 4] \cup [6, +\infty)$

80. a) $|x-2|=5 \begin{cases} x-2=5 \to x=7 \\ x-2=-5 \to x=-3 \end{cases}$ Sol: $x=-3, x=7$

b) $|x-4| \leq 7 \begin{cases} x-4=7 \to x=11 \\ x-4=-7 \to x=-3 \end{cases}$ Sol: $[-3, 11]$

c) $|x+3| \geq 6 \begin{cases} x+3=6 \to x=3 \\ x+3=-6 \to x=-9 \end{cases}$ Sol: $(-\infty,-9] \cup [3,+\infty)$

81. a) $\sqrt{x-4}$ $x-4 \geq 0$ $x \geq 4$ Sol: $[4,+\infty)$

b) $\sqrt{2x+1}$ $2x+1 \geq 0$ $x \geq -\frac{1}{2}$ Sol: $[-\frac{1}{2},+\infty)$

c) $\sqrt{-x}$ $-x \geq 0$ $x \leq 0$ Sol: $(-\infty,0]$

d) $\sqrt{3-2x}$ $3-2x \geq 0$ $\frac{3}{2} \geq x$ Sol: $(-\infty, 3/2]$

e) $\sqrt{-x-1}$ $-x-1 \geq 0$ $-1 \geq x$ Sol: $(-\infty,-1]$

f) $\sqrt{1+\frac{x}{2}}$ $1+\frac{x}{2} \geq 0$ $2+x \geq 0$ $\to x \geq -2$ Sol: $[-2,+\infty)$

82. a) $7 \text{ i } 3 \to 4$ $|7-3|=4$

b) $5 \text{ i } 11 \to 6$ $|5-11|=6$

c) $-3 \text{ i } -9 \to 6$ $|-3-(-9)|=6$

d) $-3 \text{ i } 4 \to 7$ $|-3-4|=7$

83. a) $(1,6] \cup [2,5) = (1,6]$

b) $[-1,3) \cup (0,3] = [-1,3]$

c) $(1,6] \cap [2,7) = [2,6]$

d) $[-1,3) \cap (0,4) = (0,3)$

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84. a) $\{x \mid -3 \leq x \leq 1\}$ $[-3,1]$

b) $\{x \mid 0'49 \leq x \leq 4'51\}$ $[0'49, 4'51]$

c) $\{x \mid \frac{5}{3} \leq x \leq \frac{7}{3}\}$ $[5/3, 7/3]$

85. a) $(-1,2]$ $a = \frac{-1+2}{2} = \frac{1}{2}$ $r = \left|\frac{2-(-1)}{2}\right| = \frac{3}{2}$

És un entorn de centre $\frac{1}{2}$ i radi $\frac{3}{2}$

b) $(1'3, 2'9]$ $a = \frac{1'3+2'9}{2} = 2'1$ $r = \left|\frac{2'9-1'3}{2}\right| = \frac{4}{5} = 0'8$

És un entorn de centre $2'1$ i radi $0'8$

c) $(-2'2, 0'2)$ $a = \frac{-2'2+0'2}{2} = -1$ $r = \left|\frac{0'2-(-2'2)}{2}\right| = 1'2$

És un entorn de centre -1 i radi $1'2$

d) $(-4, -2'8)$ $\quad a = \frac{-4-2'8}{2} = -3'4 \quad r = \left|\frac{-2'8-(-4)}{2}\right| = 0'6$

És un entorn de centre $-3'4$ i radi $0'6$

86. a) $|a|<b$ equival $-b<a<b$ Certa

$a<b$ i $a>-b$

b) $|-a| = -|a|$ Fals

$|-a| = a$ Ex: $|-3| = 3 \quad -|-3| = -3$

c) $|a+b| = |a| + |b|$ Fals

Ex: $|5-7| = 2 \quad |5| = 5 \quad |7| = 7 \rightarrow |5| + |7| = 5+7 = 12$

d) $|a \cdot b| = |a| \cdot |b|$ Certa

87. a) $\log_2 1024 = \log_2 2^{10} = 10 \cdot \log_2 2 = 10$

b) $\log 0{,}001 = \log 10^{-3} = -3 \cdot \log 10 = -3$

c) $\log_2 \frac{1}{64} = \log_2 \frac{1}{2^6} = \log_2 2^{-6} = -6 \cdot \log_2 2 = -6$

d) $\log_{\sqrt{3}} 3 = \log_{\sqrt{3}} (\sqrt{3})^2 = 2 \cdot \log_{\sqrt{3}} \sqrt{3} = 2$

e) $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2} \cdot \log_3 3 = \frac{1}{2}$

f) $\log_2 \sqrt{8} = \log_2 2^{3/2} = \frac{3}{2} \cdot \log_2 2 = \frac{3}{2}$

g) $\log_{1/2} \frac{1}{\sqrt{2}} = \log_{1/2} \left(\frac{1}{2}\right)^{1/2} = \frac{1}{2} \cdot \log_{1/2} \frac{1}{2} = \frac{1}{2}$

h) $\log_\pi 1 = 0$

88. a) $\log_2 64 + \log_2 \frac{1}{4} - \log_3 9 - \log_2 \sqrt{2} =$

$= \log_2 2^6 + \log_2 2^{-2} - \log_3 3^2 - \log_2 2^{1/2} =$

$= 6 + (-2) - 2 - \frac{1}{2} = 2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

b) $\log_2 \frac{1}{32} + \log_3 \frac{1}{27} - \log_2 1 =$

$= \log_2 2^{-5} + \log_3 3^{-3} - 0 = -5 - 3 - 0 = -8$

89. a) $\log_x 125 = 3 \rightarrow \log_x 5^3 = 3 \rightarrow \boxed{x = 5}$

$x^3 = 125 \; ; \; x^3 = 5^3 \rightarrow x = 5$

b) $\log_x \frac{1}{9} = -2$

$x^{-2} = \frac{1}{9} \rightarrow x^{-2} = \frac{1}{3^2} \rightarrow x^{-2} = 3^{-2} \rightarrow \boxed{x = 3}$

90. a) $\log 3^x = 2$

$x \log 3 = 2$

$x = \frac{2}{\log 3}$

$x \simeq 4{,}19$

b) $\log x^2 = -2$

$\log x^2 = \log 10^{-2}$

$x^2 = 10^{-2}$

$x^2 = \left(\frac{1}{10}\right)^2$

$x = \frac{1}{10} = 0'1$

c) $7^x = 115$

$\log_7 7^x = \log_7 115$

$x \log_7 7 = \log_7 115$

$x = \log_7 115$

$x \simeq 2'44$

d) $5^{-x} = 3$

$\log_5 5^{-x} = \log_5 3$

$-x \log_5 5 = \log_5 3$

$x = -\log_5 3$

$x \simeq -0'68$

91. a) $\log \sqrt{148} \simeq 1'085$; $10^{1'085} \simeq 12{,}1618\ldots$ (12,16552…)

b) $\ln(2{,}3 \cdot 10^{11}) \simeq 26'161$; $e^{26'161} \simeq 2'299\ldots$

c) $\ln(7'2 \cdot 10^{-5}) \simeq -9'539$; $e^{-9'539} \simeq 7'199\ldots$

d) $\log_3 42'9 \simeq 3'421$; $3^{3'421} \simeq 42'88$

e) $\log_5 1'95 \simeq 0'415$; $5^{0'415} \simeq 1'9501\ldots$

f) $\log_2 0'034 \simeq -4'878$; $2^{-4{,}878} \simeq 0'0340075\ldots$

92. a) $\log_x \frac{1}{4} = 2 \quad x^2 = \frac{1}{4} \rightarrow x^2 = \left(\frac{1}{2}\right)^2 \rightarrow x = \frac{1}{2}$

b) $\log_x 2 = \frac{1}{2} \quad x^{1/2} = 2 \rightarrow (x^{1/2})^2 = 2^2 \rightarrow x = 4$

c) $\log_x 0{,}04 = -2 \quad x^{-2} = 0{,}04 \rightarrow x^{-2} = \frac{4}{100} \rightarrow x^{-2} = \left(\frac{2}{10}\right)^2 \rightarrow x^{-2} = \left(\frac{10}{2}\right)^{-2}$

$\rightarrow x = \frac{10}{2} \rightarrow x = 5$

d) $\log_x 4 = -\frac{1}{2} \quad x^{-1/2} = 4 \rightarrow \frac{1}{x^{1/2}} = 4 \rightarrow \frac{1}{x^{1/2}} = (4^2)^{1/2} \rightarrow$

$\rightarrow \frac{1}{x^{1/2}} = \frac{1}{\left(\frac{1}{16}\right)^{1/2}} \rightarrow x = \frac{1}{16}$

93. a) $\ln x = \ln 17 + \ln 13 \rightarrow \ln x = \ln(17 \cdot 13) \rightarrow x = 17 \cdot 13 \rightarrow x = 221$

b) $\log x = \log 36 - \log 9 \rightarrow \log x = \log\left(\frac{36}{9}\right) \rightarrow x = \frac{36}{9} \rightarrow x = 4$

c) $\ln x = 3 \ln 5 \rightarrow \ln x = \ln 5^3 \rightarrow x = 5^3 \rightarrow x = 125$

d) $\log x = \log 12 + \log 25 - 2 \log 6 \rightarrow \log x = \log\left(\frac{12 \cdot 25}{6^2}\right) \rightarrow x = \frac{12 \cdot 25}{36} \rightarrow x = \frac{25}{3}$

e) $\ln x = 4\ln 2 - \frac{1}{2}\ln 25 \rightarrow \ln x = \ln\left(\frac{2^4}{\sqrt{25}}\right) \rightarrow x = \frac{2^4}{\sqrt{25}} \rightarrow x = \frac{16}{5}$

94. $\log 3 = 0'477$

$\log 30 = \log(3\cdot 10) = \log 3 + \log 10 = 0'477 + 1 = 1'477$

$\log 300 = \log(3\cdot 10^2) = \log 3 + \log 10^2 = \log 3 + 2\log 10 = 0'477 + 2 = 2'477$

$\log 3000 = \log(3\cdot 10^3) = \log 3 + \log 10^3 = \log 3 + 3\log 10 = 0'477 + 3 = 3'477$

$\log 0'3 = \log(3\cdot 10^{-1}) = \log 3 + \log 10^{-1} = \log 3 - \log 10 = 0'477 - 1 = -0'523$

$\log 0'03 = \log(3\cdot 10^{-2}) = \log 3 + \log 10^{-2} = \log 3 - 2\log 10 = 0'477 - 2 = -1'523$

$\log 0'003 = \log(3\cdot 10^{-3}) = \log 3 + \log 10^{-3} = \log 3 - 3\log 10 = 0'477 - 3 = -2'523$

95. $\log k = 14'4$

a) $\log\left(\frac{k}{100}\right) = \log k - \log 100 = \log k - \log 10^2 = 14'4 - 2 = 12'4$

b) $\log(0'1k^2) = \log 0'1 + \log k^2 = \log 10^{-1} + 2\log k = -1 + 2\cdot 14'4 = 27'8$

c) $\log\sqrt[3]{\frac{1}{k}} = \log k^{-1/3} = -\frac{1}{3}\cdot\log k = -\frac{1}{3}\cdot 14'4 = -4'8$

d) $(\log k)^{1/2} = (14,4)^{1/2} = \sqrt{14,4} = 3'79$

96. $\ln k = 0'45$

a) $\ln\left(\frac{k}{e}\right) = \ln k - \ln e = 0'45 - 1 = -0'55$

b) $\ln\sqrt[3]{k} = \frac{\ln k}{3} = \frac{0'45}{3} = 0'15$

c) $\ln\left(\frac{e^2}{k}\right) = \ln e^2 - \ln k = 2 - 0'45 = 1'55$

97. a) $x^{2'7} = 19$

$\log x^{2'7} = \log 19$

$2'7\log x = \log 19$

$\log x = \frac{\log 19}{2'7}$

$\log x = 0'47$

$x = 10^{0'47} = 2'95$

b) $\log_7 3x = 0,5$

$7^{0'5} = 3x$

$2'65 = 3x$

$0'88 = x$

c) $3^{2+x} = 172$

$\log_3 3^{2+x} = \log_3 172$

$(2+x)\log_3 3 = \log_3 172$

$2 + x = \log_3 172$

$x = \log_3(172) - 2$

$x = 2'69$

98. $\log k = x$

a) $\log k^2 = 2\log k = 2x$

b) $\log \frac{k}{100} = \log k - \log 100 = \log k - \log 10^2 = x - 2$

c) $\log \sqrt{10k} = \frac{\log(10k)}{2} = \frac{\log 10 + \log k}{2} = \frac{1+x}{2}$

99. $\frac{\log\left(\frac{1}{a}\right) + \log \sqrt{a}}{\log a^3} = \frac{\log 1 - \log a + \frac{\log a}{2}}{3\log a} = \frac{0 - \log a + \frac{\log a}{2}}{3\log a} =$

$a \neq 0$

$= \frac{\left(-1 + \frac{1}{2}\right)\log a}{3\log a} \overset{a \neq 1, a \neq 0}{=} \frac{-\frac{1}{2}}{3} = -\frac{1}{6}$

$\uparrow \log a \neq 0$

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100 a) $(3+2i)(2-i) - (1-i)(2-3i) = 6 - 3i + 4i + 2 - [2 - 3i - 2i - 3] =$

$= 8 + i - [-1 - 5i] = 8 + i + 1 + 5i = 9 + 6i$

b) $3 + 2i(-1+i) - (5-4i) = 3 - 2i - 2 - 5 + 4i = -4 + 2i$

c) $-2i - (4-i)5i = -2i - 20i - 5 = -5 - 22i$

d) $(4-3i)(4+3i) - (4-3i)^2 = 16 + 9 - (16 - 24i - 9) = 16 + 9 - 16 + 24i + 9 =$

$= 18 + 24i$

101. a) $\frac{(3+3i)(4-2i)}{2-2i} = \frac{(12 - 6i + 12i + 6)}{2-2i} = \frac{18+6i}{2-2i} \cdot \frac{2+2i}{2+2i} = \frac{36 + 36i + 12i - 12}{4+4} =$

$= \frac{24 + 48i}{8} = 3 + 6i$

b) $\frac{-2+3i}{(4+2i)(-1+i)} = \frac{-2+3i}{-4+4i-2i-2} = \frac{-2+3i}{-6+2i} \cdot \frac{2i+6}{2i+6} = \frac{-4i - 12 - 6 + 18i}{-4-36} =$

$= \frac{14i - 18}{-40} = \frac{7i - 9}{-20} = \frac{9}{20} - \frac{7}{20}i$

c) $\frac{2+5i}{3-2i}(1-i) = \frac{2 - 2i + 5i + 5}{3-2i} = \frac{7+3i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{21 + 14i + 9i - 6}{9+4} =$

$= \frac{15 + 23i}{13} = \frac{15}{13} + \frac{23}{13}i$

d) $\frac{1+i}{2-i} + \frac{-3-2i}{1+3i} = \frac{1+3i}{5} + \frac{-9+7i}{10} = \frac{2 + 6i - 9 + 7i}{10} = \frac{-7+13i}{10} = \frac{-7}{10} + \frac{13}{10}i$

$\frac{1+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{2 + i + 2i - 1}{4+1} = \frac{1+3i}{5}$ $\quad \frac{-3-2i}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{-3 + 9i - 2i - 6}{1+9} = \frac{-9+7i}{10}$

102. a) $(1-i)(4-2i)(1+3i) = (4-2i-4i-2)(1+3i) = (2-6i)(1+3i) = 2+6i-6i+18 = 20$

b) $\frac{1+2i}{2-i}(2+i) + \frac{1-2i}{2+i}(2-i) = \frac{2+i+4i-2}{2-i} + \frac{2-i-4i-2}{2+i} = \frac{5i}{2-i} + \frac{-5i}{2+i} =$

$\frac{5i(2+i)}{(2-i)(2+i)} - \frac{5i(2-i)}{(2+i)(2-i)} = \frac{10i-5}{4+1} - \frac{10i+5}{4+1} = \frac{10i-5}{5} - \frac{10i+5}{5} = \frac{-10}{5} = -2$

c) $\frac{2-i}{3-i} - \frac{1}{5}\left(\frac{1+8i}{1+3i}\right) = \frac{7-i}{10} - \frac{1}{5}\left(\frac{25+5i}{10}\right) = \frac{7-i}{10} - \frac{5+i}{10} = \frac{2-2i}{10} = \frac{1}{5} - \frac{1}{5}i$

$\frac{1+8i}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{1-3i+8i+24}{1+9} = \frac{25+5i}{10}$ $\quad \frac{2-i}{3-i} \cdot \frac{3+i}{3+i} = \frac{6+2i-3i+1}{9+1} = \frac{7-i}{10}$

d) $\frac{(2+i)^2+(1-i)^2}{1-(3/2)i} = \frac{4+4i-1+1-2i-1}{1-\frac{3}{2}i} = \frac{3+2i}{1-\frac{3}{2}i} = 2\cdot\left(\frac{3+2i}{2-3i} \cdot \frac{2+3i}{2+3i}\right) =$

$= 2 \cdot \frac{6+9i+4i-6}{4+9} = 2 \cdot \frac{13i}{13} = 2i$

e) $\frac{2-2i}{i} + \frac{3-5i}{2-i} = \frac{2-2i}{i} \cdot \frac{-i}{-i} + \frac{3-5i}{2-i} \cdot \frac{2+i}{2+i} = \frac{2i+2}{-1} + \frac{6+3i-10i+5}{4+1} =$

$= -2i-2 + \frac{11-7i}{5} = \frac{-10i-10+11-7i}{5} = \frac{1-17i}{5} = \frac{1}{5} - \frac{17}{5}i$

103. a) $i^{37} = i^{4\cdot 9+1} = i^{4\cdot 9} \cdot i = 1 \cdot i = i$ b) $i^{126} = i^{31\cdot 4+2} = i^2 = -1$

c) $i^{87} = i^{21\cdot 4+3} = i^3 = -i$ d) $i^{64} = i^{16\cdot 4} = 1$

e) $i^{-216} = \frac{1}{i^{216}} = \frac{1}{i^{54\cdot 4}} = \frac{1}{1} = 1$

104. $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

a) $1+z+z^2 = 0 \to 1 + \frac{-1+\sqrt{3}i}{2} + \left(\frac{-1+\sqrt{3}i}{2}\right)^2 = 1 + \frac{-1+\sqrt{3}i}{2} + \frac{1-3-2\sqrt{3}i}{4} =$

$= \frac{4-2+2\sqrt{3}i+1-3-2\sqrt{3}i}{4} = \frac{0}{4} = 0$

b) $\frac{1}{z} = z^2 \to \frac{1}{-\frac{1}{2} + \frac{\sqrt{3}i}{2}} = \frac{2}{-1+\sqrt{3}i} \cdot \frac{+1+\sqrt{3}i}{+1+\sqrt{3}i} = \frac{+2+2\sqrt{3}i}{1+3} = \frac{+2+2\sqrt{3}i}{4} =$

$= \frac{-1-\sqrt{3}i}{2} = z^2$

$z^2 = \frac{-2-2\sqrt{3}i}{4} = \frac{-1-\sqrt{3}i}{2}$

105. $(2+mi) + (m+5i) = 7-2i$

$(2+mi) + (m+5i) \Rightarrow (2+m) + (m+5) = 7-2i$ (Part real / Part imaginaria)

$\begin{cases} 2+m = 7 \to m = 7-2 \to m = 5 \\ m+5 = -2 \to m = -2-5 \to m = -7 \end{cases}$

106. $\frac{k+i}{1+i} = 2-i \Rightarrow \frac{1+k}{2} = 2 \to 1+k=4 \to k=3$ · $\frac{1-k}{2} = -1 \to 1-k=-2 \to k=3$

$\frac{k+i}{1+i} \cdot \frac{1-i}{1-i} = \frac{k-ki+i+1}{1+1} = \frac{(1+k)+(1-k)i}{2} = \frac{1+k}{2} + \frac{1-k}{2}i$

Per tant $k=3$.

107. $(a+bi)^2 = 3+4i \Rightarrow (a^2+b^2) + 2abi = 3+4i \begin{cases} a^2-b^2=3 \to \circledast \\ 2ab=4 \to ab=2 \to a=\frac{2}{b} \end{cases}$

$(a+bi)^2 = a^2 + 2abi - b^2 = (a^2-b^2) + 2abi$

$\circledast \left(\frac{2}{b}\right)^2 - b^2 = 3 \to \frac{4}{b^2} - b^2 = 3 \to 4 - b^4 = 3b^2 \to -b^4 - 3b^2 + 4 = 0$

$\to b^2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-1)\cdot 4}}{2(-1)} = \frac{3 \pm \sqrt{9+16}}{-2} = \frac{3 \pm \sqrt{25}}{-2} = \frac{3 \pm 5}{-2} = \begin{cases} b^2 = -4 \\ b^2 = +1 \end{cases}$

· Si $b^2 = 1 \to b = \pm\sqrt{1} \to b = \pm 1$

· Si $b^2 = -4 \to b = \pm\sqrt{-4}$ (b ha de ser un nombre real, no val)

Llavors

Si $b=1 \Rightarrow a = \frac{2}{1} = 2 \Rightarrow$ **$a=2$ i $b=1$**

Si $b=-1 \Rightarrow a = \frac{2}{-1} = -2 \Rightarrow$ **$a=-2$ i $b=-1$**

108. $(2-ai)(3-bi) = 8+4i$

$6 - 2bi - 3ai - ab = 8+4i$

$(6-ab) + (-2b-3a)i = 8+4i \to \begin{cases} 6-ab=8 \to \circledast \\ -2b-3a=4 \to a = \frac{-4-2b}{3} \end{cases}$

$\circledast \; 6 - \left(\frac{-4-2b}{3}\right)\cdot b = 8 \to 6 + \frac{4b+2b^2}{3} = 8 \to 18 + 4b + 2b^2 = 24 \to$

$\to 2b^2 + 4b - 6 = 0 \to b^2 + 2b - 3 = 0 \to b = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot(-3)}}{2\cdot 1} = \frac{-2 \pm 4}{2} =$

$= \begin{cases} b=-3 \\ b=1 \end{cases}$

· Si $b=-3 \Rightarrow a = \frac{-4-2(-3)}{3} = \frac{+2}{3} \Rightarrow$ **$a=\frac{2}{3}$ i $b=3$**

· Si $b=-1 \Rightarrow a = \frac{-4-2\cdot(-1)}{3} = \frac{-6}{3} = -2 \Rightarrow$ **$a=-2$ i $b=-1$**

109. $a-3i = \frac{2+bi}{5-3i}$ $\quad \frac{2+bi}{5-3i} \cdot \frac{5+3i}{5+3i} = \frac{10+6i+5bi-3b}{25+9} = \frac{(10-3b)+(6+5b)i}{34}$

$34a - 102i = (10-3b) + (6+5b)i \begin{cases} 34a = 10-3b \to \boxed{a = \frac{10-3\cdot\left(\frac{-102}{5}\right)}{34} = \frac{50+324}{170} = \frac{374}{170} = \frac{11}{5}} \\ -102 = 6+5b \to \boxed{b = \frac{-102-6}{5} = \frac{-108}{5}} \end{cases}$

110. $(3-6i)(4+bi) = 12+3bi-24i+6b = (12+6b)+(3b-24)i$

a) Imaginari pur: ⇒ la part real ha de ser 0.

$12+6b=0 \rightarrow b=\frac{-12}{6} \rightarrow \boxed{b=-2}$

b) Real ⇒ la part imaginaria ha de ser 0.

$3b-24=0 \rightarrow b=\frac{24}{3} \rightarrow \boxed{b=8}$

111. $(a-2i)^2 = a^2-4-4ai = (a^2-4)+(-4a)i$

Imaginari pur ⇒ la part real ha de ser 0.

$a^2-4=0 \rightarrow a^2=4 \rightarrow a=\pm\sqrt{4} \rightarrow \boxed{a_1=2,\ a_2=-2}$

112. $(x+2+ix)(x-i) = x^2-xi+2x-2i+ix^2+x = (x^2+3x)+(-x-2+x^2)i$

$= (x^2+3x)+(x^2-x-2)i$

Nombre real ⇒ la part imaginaria ha de ser 0.

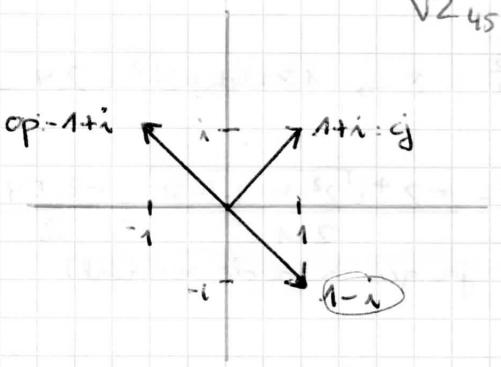
$x^2-x-2=0 \rightarrow x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot1\cdot(-2)}}{2\cdot1}=\frac{1\pm3}{2}=\begin{cases}\boxed{x_1=2}\\ \boxed{x_2=-1}\end{cases}$

113. a) $1-i = \sqrt{2}_{315^\circ}$ $\quad r=\sqrt{1^2+(-1)^2}=\sqrt{2}$

Oposat: $-1+i=\sqrt{2}_{315^\circ+180^\circ}=\sqrt{2}_{135^\circ}$ $\quad tg\,\alpha=\frac{-1}{1}\rightarrow\alpha=-45^\circ$

Conjugat: $1+i=\sqrt{2}_{360^\circ-315}$ $\quad 495^\circ=360^\circ+135^\circ$ $\quad \beta=360^\circ-45^\circ=315^\circ$

$=\sqrt{2}_{45^\circ}$ $\quad$ (una volta)



b) $-1+i = \sqrt{2}_{135^\circ}$

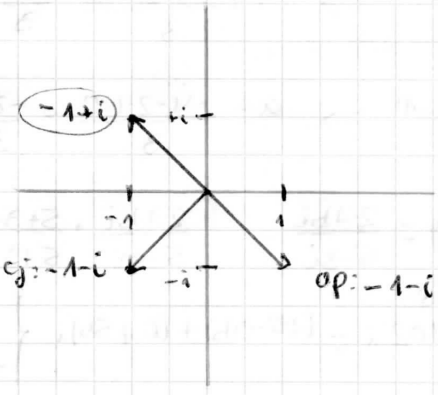
Oposat: $1-i=\sqrt{2}_{135^\circ+180^\circ}=\sqrt{2}_{315^\circ}$

Conjugat: $-1-i=\sqrt{2}_{360^\circ-135^\circ}=\sqrt{2}_{225^\circ}$

$r=\sqrt{(-1)^2+1^2}=\sqrt{2}$

$tg\,\alpha=\frac{1}{-1}\rightarrow\alpha=-45^\circ$

$\beta=-45^\circ+180^\circ=135^\circ$



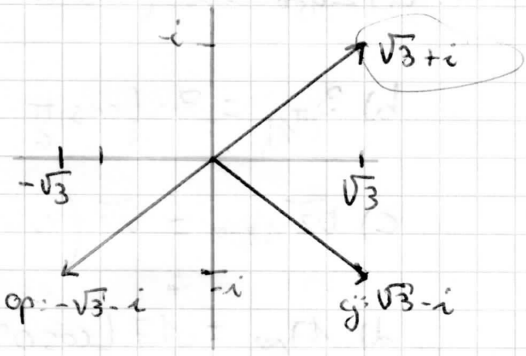
c) $\sqrt{3}+i$

oposat: $-\sqrt{3}-i = 2_{30°+180°} = 2_{210°}$

conjugat: $\sqrt{3}-i = 2_{-30°+360°} = 2_{330°}$

$r = \sqrt{\sqrt{3}^2+1^2} = \sqrt{4} = 2$

$\operatorname{tg}\alpha = \frac{1}{\sqrt{3}} \quad \alpha = 30°$



d) $-\sqrt{3}-i = 2_{210°}$

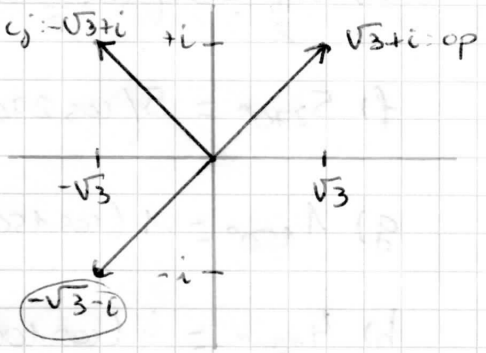
oposat: $\sqrt{3}+i = 2_{210+180°} = 2_{390°} = 2_{30°}$

conjugat: $-\sqrt{3}+i = 2_{360°-210°} = 2_{150°}$

$r = \sqrt{(-\sqrt{3})^2+(-1)^2} = \sqrt{4} = 2$

$\operatorname{tg}\alpha = \frac{-1}{-\sqrt{3}} \rightarrow \alpha = 30°$

$\beta = 180°+30° = 210°$

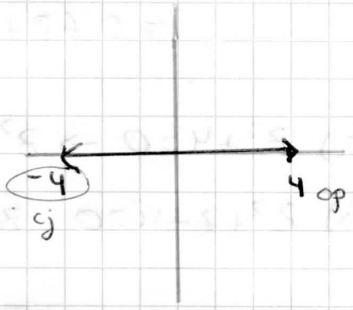


e) $-4 = 4_{180°}$

Oposat: $4 = 4_{0°}$

Conjugat: $-4 = 4_{180°}$

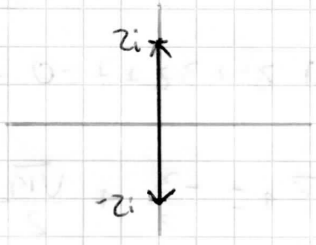
$r = \sqrt{(-4)^2+0^2} = 4$



f) $2i = 2_{90°}$

Oposat: $-2i = 2_{270°}$

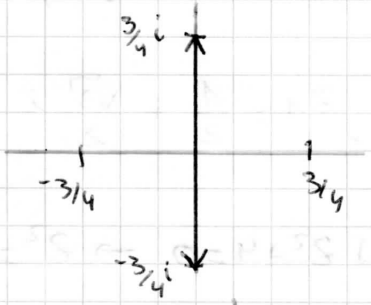
Conjugat: $-2i = 2_{270°}$



g) $-\frac{3}{4}i = \left(\frac{3}{4}\right)_{270°}$

Oposat: $\frac{3}{4}i = \left(\frac{3}{4}\right)_{90°}$

Conjugat: $\frac{3}{4}i = \left(\frac{3}{4}\right)_{90°}$

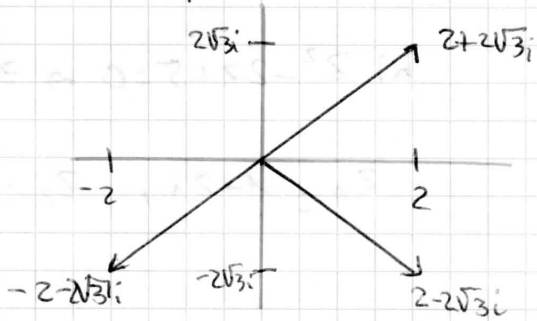


h) $2+2\sqrt{3}i = 4_{60°}$

Oposat: $-2-2\sqrt{3}i = 4_{60°+180°} = 4_{240°}$

Conjugat: $2-2\sqrt{3}i = 4_{360°-60°} = 4_{300°}$

$r = \sqrt{2^2+(2\sqrt{3})^2} = 4 \quad \operatorname{tg}\alpha = \frac{2\sqrt{3}}{2} = 60°$



114. a) $2_{45°} = 2\cdot(\cos 45° + i \sin 45°) = 2\cdot\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{2} + \sqrt{2}i$

b) $3_{(\pi/6)} = 3\cdot\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

c) $\sqrt{2}_{180°} = \sqrt{2}\left(\cos 180° + i\sin 180°\right) = \sqrt{2}(-1 + i\,0) = -\sqrt{2}$

d) $17_{00} = 17(\cos 0° + i\sin 0°) = 17(1 + i\cdot 0) = 17$

e) $1_{\frac{\pi}{2}} = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 0 + i\cdot 1 = i$

f) $5_{270°} = 5(\cos 270° + i\sin 270°) = 5\cdot(0 + i(-1)) = -5i$

g) $1_{150°} = 1(\cos 150° + i\sin 150°) = 1\cdot\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

h) $4_{100°} = 4(\cos 100° + i\sin 100°) = 4\cdot(-0{,}17 + i\cdot 0{,}98) =$
$= 0{,}68 + i\,3{,}92i$

115. a) $z^2 + 4 = 0 \to z^2 = -4 \to z = \pm\sqrt{-4} \to z_1 = 2i,\ z_2 = -2i$

b) $z^2 + z + 4 = 0 \quad z = \frac{-1 \pm \sqrt{1^2 - 4\cdot 1\cdot 4}}{2\cdot 1} = \frac{-1 \pm \sqrt{-15}}{2} = \begin{cases} \frac{-1+\sqrt{-15}}{2} \\ \frac{-1-\sqrt{-15}}{2} \end{cases}$

$z_1 = \frac{-1}{2} + \frac{\sqrt{15}}{2}i,\quad z_2 = \frac{-1}{2} - \frac{\sqrt{15}}{2}i$

c) $z^2 + 3z + 7 = 0 \to z = \frac{-3 \pm \sqrt{3^2 - 4\cdot 1\cdot 7}}{2\cdot 1} = \frac{-3 \pm \sqrt{-19}}{2} = \begin{cases} \frac{-3+\sqrt{-19}}{2} \\ \frac{-3-\sqrt{-19}}{2} \end{cases}$

$z_1 = \frac{-3}{2} + \frac{\sqrt{19}}{2}i,\quad z_2 = \frac{-3}{2} - \frac{\sqrt{19}}{2}i$

d) $z^2 - z + 1 = 0 \to z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\cdot 1\cdot 1}}{2\cdot 1} = \frac{1 \pm \sqrt{-3}}{2} = \begin{cases} \frac{1+\sqrt{-3}}{2} \\ \frac{1-\sqrt{-3}}{2} \end{cases}$

$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i,\quad z_2 = \frac{1}{2} - \frac{\sqrt{3}}{2}$

116. a) $z^2 + 4 = 0 \to z^2 = -4 \to z = \pm\sqrt{-4} = \pm 2i \begin{cases} \to z_1 = 2i \\ \to z_2 = -2i \end{cases}$

b) $z^2 - 2z + 5 = 0 \to z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot 1\cdot 5}}{2\cdot 1} = \frac{2 \pm \sqrt{-16}}{2} = \begin{cases} \frac{2+4i}{2} \\ \frac{2-4i}{2} \end{cases}$

$z_1 = 1 + 2i,\quad z_2 = 1 - 2i$

c) $2z^2+10=0 \to z^2=-\frac{10}{2} \to z^2=-5 \to z=\pm\sqrt{-5}$

$z_1=-\sqrt{5}\,i \quad , \quad z_2=+\sqrt{5}\,i$

d) $z^4+13z^2+36=0 \to z^2=\frac{-13\pm\sqrt{13^2-4\cdot1\cdot36}}{2\cdot1}=\frac{-13\pm\sqrt{25}}{2}=\frac{-13\pm5}{2}=\begin{cases} z^2=-9 \\ z^2=-4 \end{cases}$

· Si $z^2=-9 \to z=\pm\sqrt{-9}$ $\begin{cases} z_1=-3i \\ z_2=3i \end{cases}$

· Si $z^2=-4 \to z=\pm\sqrt{-4}$ $\begin{cases} z_3=-2i \\ z_4=2i \end{cases}$

Pàg 40

117. $18:(6{,}02\cdot10^{23})=2{,}99\cdot10^{-23}$ grams

$2{,}99\cdot10^{-23}$ grams fa una molècula d'aigua.

118. $R=\frac{\rho l}{s} \to l=\frac{R\cdot s}{\rho}$

$l=\frac{20\cdot\pi\cdot\left(\frac{0'003}{2}\right)^2}{1'7\cdot10^{-8}}=8315'98$ m

l = longitud $\quad \rho=1'7\cdot10^{-8}\,\Omega\cdot m$

s = secció del fil

$d=3mm=0'003$ m

119. $v=\sqrt{\frac{2\cdot6'67\cdot10^{-11}\cdot5'98\cdot10^{24}}{6{,}37\cdot10^{6}}}\simeq11190{,}74\ m/s$

120. $\sqrt{6+\sqrt{27}}\cdot\sqrt{6-\sqrt{27}}=\sqrt{(6+\sqrt{27})\cdot(6-\sqrt{27})}=\sqrt{6^2-\sqrt{27}^2}=\sqrt{36-27}=\sqrt{9}=3$

121. a) $\sqrt{a^3}-2a\sqrt[4]{a^2}+3a\sqrt[6]{a^3}-\sqrt[8]{a^{12}}=a\sqrt{a}-2a\sqrt{a}+3a\sqrt{a}-\sqrt{a^3}=$

$=a\sqrt{a}-2a\sqrt{a}+3a\sqrt{a}-a\sqrt{a}=a\sqrt{a}$

b) $\frac{\sqrt{98}-\sqrt{18}}{\sqrt{96}}\cdot30\sqrt{3}=\frac{\sqrt{2\cdot7^2}-\sqrt{2\cdot3^2}}{\sqrt{2^5\cdot3}}\cdot30\sqrt{3}=\frac{7\sqrt{2}-3\sqrt{2}}{2^2\sqrt{2}\sqrt{3}}\cdot30\sqrt{3}=$

$=\frac{4\sqrt{2}\cdot30}{4\sqrt{2}}=30$

c) $(\sqrt{2}+\sqrt{3})(\sqrt{6}-1)=\sqrt{12}-\sqrt{2}+\sqrt{18}-\sqrt{3}=2\sqrt{3}-\sqrt{2}+3\sqrt{2}-\sqrt{3}=$

$=\sqrt{3}+2\sqrt{2}$

122. a) $\frac{1}{\sqrt{2}-1}-\frac{1}{\sqrt{2}+1}=\frac{1\cdot(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}-\frac{1\cdot(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{\sqrt{2}+1}{2-1}-\frac{\sqrt{2}-1}{2-1}=\frac{\sqrt{2}+1}{1}-\frac{\sqrt{2}-1}{1}=2$

b) $\frac{7}{3-\sqrt{2}}-\frac{1}{\sqrt{3}-\sqrt{2}}+\frac{1}{2-\sqrt{3}}=\frac{7(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}-\frac{1\cdot(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}+\frac{1\cdot(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}=\frac{7(3+\sqrt{2})}{9-2}-\frac{\sqrt{3}+\sqrt{2}}{3-2}+$

$+\frac{2+\sqrt{3}}{4-3}=\frac{7(3+\sqrt{2})}{7}-\frac{\sqrt{3}+\sqrt{2}}{1}+\frac{2+\sqrt{3}}{1}=3+\sqrt{2}-\sqrt{3}-\sqrt{2}+2+\sqrt{3}=5$

c) $\frac{5}{\sqrt{6}} + \frac{2}{\sqrt{6}+3\sqrt{2}} - \frac{4\sqrt{2}}{\sqrt{3}} = \frac{5\sqrt{6}}{\sqrt{6}\sqrt{6}} + \frac{2(\sqrt{6}-3\sqrt{2})}{(\sqrt{6}+3\sqrt{2})(\sqrt{6}-3\sqrt{2})} \overset{\frac{4\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}}}{=} \frac{5\sqrt{6}}{6} + \frac{2\sqrt{6}-6\sqrt{2}}{6-9\cdot 2} - \frac{4\sqrt{6}}{3} =$

$= \frac{5\sqrt{6}}{6} + \frac{2\sqrt{6}-6\sqrt{2}}{-12} - \frac{4\sqrt{6}}{3} = \frac{5\sqrt{6}}{6} + \frac{-2\sqrt{6}+6\sqrt{2}}{12} - \frac{4\sqrt{6}}{3} = \frac{10\sqrt{6}}{12} + \frac{-2\sqrt{6}+6\sqrt{2}}{12} -$

$- \frac{16\sqrt{6}}{12} = \frac{10\sqrt{6}-2\sqrt{6}+6\sqrt{2}-16\sqrt{6}}{12} = \frac{-8\sqrt{6}+6\sqrt{2}}{12} = \frac{-2\sqrt{6}}{3} + \frac{\sqrt{2}}{2}$

123. a) Cert b) Fals c) Cert d) Cert e) Cert f) Fals g) Fals
h) Cert i) Cert j) Fals

124. a) Cert b) Fals c) Fals d) Cert

125. a) $(-27)^{\frac{1}{3}} \leftarrow \overset{3}{(-3)} \leftarrow -9$ b) $4^{-\frac{1}{2}} \leftarrow \overset{1/\sqrt{2}}{(2^{-1})} \leftarrow -2$

126. $\log 348$

$10^2 = 100$ $10^3 = 1000$ $\rightarrow 10^2 < 348 < 10^3$

$2\log 10 < \log_{10} 348 < 3\log 10$

Entre 2 i 3. $2 < \log_{10} 348 < 3$

127. $\log x = a$

$\log\left(\frac{1}{x}\right) = \log 1 - \log x = 0 - a = -a$

128. a) $\log m + \log n = \log(m+n)$ fals. $\log m + \log n = \log(m \cdot n)$

b) $\log m + \log n = \log(m \cdot n)$ Certa (Propietat dels logarítmes)

c) $\log m - \log n = \frac{\log m}{\log n}$ fals. $\log m - \log n = \log\left(\frac{m}{n}\right)$

d) $\log m - \log n = \log\left(\frac{m}{n}\right)$ Certa (Propietat dels logarítmes)

e) $\log x^2 = \log x + \log x$ Cert. $\log x^2 = \log(x \cdot x) \overset{\text{prop}}{=} \log x + \log x$

f) $\log(a^2-b^2) = \log(a+b) + \log(a-b)$ Certa $\log(a^2-b^2) = \log[(a-b)(a+b)] =$

$= \log(a-b) + \log(a+b)$

Pàg 41

129. $n \neq 0$, $n \in \mathbb{N}$

a) $\frac{n}{2}$ n parell b) $\frac{3}{n}$ $n=1$ o $n=3$ c) $n-5$ $\forall n \in \mathbb{N}$

d) $n + \frac{1}{2}$ Cap e) $\sqrt{n}$ n quadrat perfecte

130. $\log a = 1 + \log b \to \log a = \log 10 + \log b \to \log a = \log(10b)$

$\to a = 10b$

131. $\log a + \log b = 0 \to \log a + \log b = \log 1 \to \log(ab) = \log 1 \to ab = 1$

$\to a = \frac{1}{b}$

132. a) $m \cdot n > 0$ i $m + n > 0$ $\quad m > 0$ i $n > 0$

b) $m \cdot n > 0$ i $m + n < 0$ $\quad m < 0$ i $n < 0$

c) $m \cdot n < 0$ i $m - n > 0$ $\quad m > 0$ i $n < 0$

d) $m \cdot n < 0$ i $m - n < 0$ $\quad m < 0$ i $n > 0$

133. $\log_a(P \cdot Q) = \log_a P + \log_a Q$

$\log_a P = p \xrightarrow{\text{Per definició de logaritme}} a^p = P$

$\log_a Q = q \xrightarrow{\text{Per definició de logaritme}} a^q = Q$

$\Big\} \xrightarrow{\text{multipliquem les igualtats}} P \cdot Q = a^p \cdot a^q \to P \cdot Q = a^{p+q}$

$\to \log_a(P \cdot Q) = (p+q)\log_a \to \log_a(P \cdot Q) = p + q \to \log_a(P \cdot Q) = \log_a P + \log_a Q$

134. $\log_a(P/Q) = \log_a P - \log_a Q$

$\log_a P = p \to a^p = P$

$\log_a Q = q \to a^q = Q$

$\Big\} \to \frac{P}{Q} = \frac{a^p}{a^q} \to \frac{P}{Q} = a^{p-q} \to \log_a\left(\frac{P}{Q}\right) = \log_a a^{p-q}$

$\to \log_a\left(\frac{P}{Q}\right) = (p-q)\log_a a \to \log_a\left(\frac{P}{Q}\right) = p - q \to \log_a\left(\frac{P}{Q}\right) = \log_a P - \log_a Q$

135. $\log_a P^n = n \log_a P$

$\log_a P = p \to a^p = P \to a^p \cdot \overset{n}{\cdots} \cdot a^p = P \cdot \overset{n}{\cdots} \cdot P \to a^{n \cdot p} = P^n \to$

$\to \log_a a^{np} = \log_a P^n \to np \log_a a = \log_a P^n \to np = \log_a P^n$

136. $\log_a \sqrt[n]{P} = \frac{\log_a P}{n}$

$\log_a P = p \to a^p = P \to (a^p)^{1/n} = P^{1/n} \to a^{\frac{p}{n}} = P^{1/n} \to \log_a a^{p/n} = \log_a P^{1/n}$

$\to \frac{p}{n}\log_a a = \log_a P^{1/n} \to \frac{p}{n} = \log_a P^{1/n} \to \frac{\log_a P}{n} = \log_a \sqrt[n]{P}$

137. $\log_a P = \log P / \log a$

$\log_a P = p \to a^p = P \to \log a^p = \log P \to p \log a = \log P \to p = \frac{\log P}{\log a}$

$\to \log_a P = \frac{\log P}{\log a}$

138. $x \in \mathbb{N}$, $x > 1$

$$\frac{1}{x+1} \; , \; x \; , \; \frac{1}{x} \; , \; \frac{-1}{x} \; , \; \frac{1}{-x-1}$$

$$\frac{1}{-x-1} < \frac{1}{x+1} < \frac{1}{x} < x$$

139. $a, \; a^2, \; \frac{1}{a}, \; \sqrt{a}$

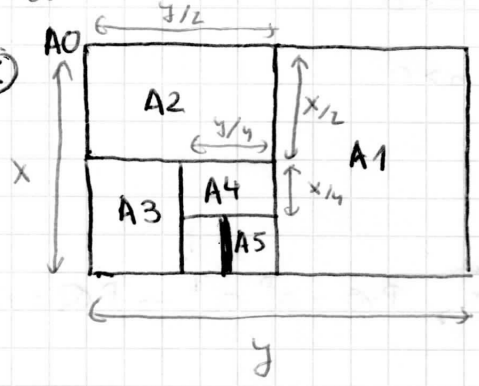
I) Si $a > 1$

$$\frac{1}{a} < \sqrt{a} < a < a^2$$

II) Si $0 < a < 1$

$$a^2 < a < \sqrt{a} < \frac{1}{a}$$

140. (I)



$AO = 1\,m^2 \qquad x \cdot y = 1 \rightarrow y = \frac{1}{x}$ (*)

A1 : AO i A1 són semblants :

$$\frac{y}{x} = \frac{x}{y/2} \rightarrow y^2 = 2x^2$$

(*) $\left(\frac{1}{x}\right)^2 = 2x^2 \rightarrow \frac{1}{x^2} = 2x^2 \rightarrow 1 = 2x^4$

$$\rightarrow x^4 = \frac{1}{2} \rightarrow x = \sqrt[4]{\frac{1}{2}} = \frac{1}{\sqrt[4]{2}}$$

$$y = \frac{1}{1/\sqrt[4]{2}} = \sqrt[4]{2}$$

dim. AO : llarg : $\sqrt[4]{2}$ m i ample : $\frac{1}{\sqrt[4]{2}}$ m

dim A4 : llarg : $y/4 \rightarrow \frac{\sqrt[4]{2}}{4} = 0'297\,m = 297\,mm$

ample : $\frac{x}{4} \rightarrow \frac{1/\sqrt[4]{2}}{4} = 0'210\,m = 210\,mm$

(II)

La raó entre els costats del rectangle AO $\rightarrow \frac{y}{x} = \frac{\sqrt[4]{2}}{1/\sqrt[4]{2}} = (\sqrt[4]{2})^2 = \sqrt{2}$

i és la mateixa que A1 $\rightarrow \frac{x}{y/2} = \frac{1/\sqrt[4]{2}}{\sqrt[4]{2}/2} = \frac{2}{(\sqrt[4]{2})^2} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

i és la mateixa que A2 $\rightarrow \frac{y/2}{x/2} = \frac{y}{x} = \sqrt{2}$

i és la mateixa que A3 $\rightarrow \frac{x/2}{y/4} = \frac{x}{y/2} = \sqrt{2}$

És la mateixa en tots ja que tots aquests rectangles són semblants.

La raó entre els costats MNPQ és

$$\frac{\overline{MN}}{\overline{MQ}} \quad \frac{y+x}{x} = \frac{y/x + x/x}{x/x} = \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$$

Si MRQS és semblant a MNPQ cal que compleixi $\frac{\overline{MQ}}{\overline{MR}} = \sqrt{2}+1$

$$\frac{\overline{MQ}}{\overline{MR}} = \frac{x}{y-x} = \frac{x/x}{y/x - x/x} = \frac{1}{y/x - 1} = \frac{1}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} =$$

$$= \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$$

Per tant MRQS i MNPQ són semblants.

141. De la pàgina 1 a la 9 → 9 dígits

de la pàgina 10 a la 99 → 90 pàgines · 2 dígits/pàgina = 180 dígits

de la pàgina 100 a la 999 → 900 pàgines · 3 dígits/pàgina = 2700 dígits

9 + 180 + 2700 = 2889 dígits

2993 − 2889 = 104 dígits on van 4 dígits a cada pàgina

104 : 4 = 26 pàgines més.

999 pàgines + 26 pàgines = 1024 pàgines

El llibre té 1024 pàgines.